# Integrating Expanders into Heat Exchanger Networks above Ambient Temperature

**Chao Fu and Truls Gundersen**
Dept. of Energy and Process Engineering, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

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*The integration of expanders into heat exchanger networks (HENs) is a complex task since both heat and work are involved. In addition, the role of streams (as hot or cold), the utility demand, and the location of pinch points may change. With certain well-defined conditions, four theorems are proposed for the integration of expanders into HENs above ambient temperature with the objective of minimizing exergy consumption. A straightforward graphical methodology for above ambient HENs design including expanders is developed on the basis of Grand Composite Curves (GCCs). It is concluded that to achieve a design with minimum exergy consumption, stream splitting may be applied and expansion should be done at pinch temperatures, hot utility temperature, or ambient temperature. In the majority of cases, however, and in line with the concept of Appropriate Placement from Pinch Analysis, expansion at pinch temperatures gives the minimum exergy consumption.*  *AIChE J*, 61: 3404–3422, 2015
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## Introduction

The synthesis of heat exchanger networks (HENs) has achieved a notable success in the past decades. One of the well-known methodologies for HEN synthesis is Pinch Analysis and the engineer-driven Pinch Design Method.[1] For a given set of hot and cold streams, the pinch temperatures are fixed by the minimum temperature difference ($\Delta T_{\min}$) for heat transfer. Heat exchange takes place separately in the above and below pinch temperature regions. Any heat transfer across the pinch increases both hot and cold utilities. Comprehensive reviews on HEN synthesis are provided by Gundersen and Naess[2] and Furman and Sahinidis.[3] The heat integration problem is also extended to include other equipment such as reactors,[4] distillation columns,[5] evaporators,[6] heat pumps, and heat engines.[7] In all the above cases,[4–7] only heat (no work) is integrated between these types of equipment and the process streams of the remaining (often referred to as the "background") process.

The concept of Appropriate Placement[7] (also referred to as Correct Integration) is a special case of the plus/minus principle,[1,8] and it addresses how different units of equipment can be integrated with heat recovery processes to ensure it results in energy savings. A quantitative approach to Appropriate Placement is based on the Grand Composite Curve (GCC) and allows the calculation of the amount of heat that can be correctly integrated.[7] While this type of analysis is simple for distillation columns, evaporators, heat pumps, and heat engines, it is considerably more complicated for pressure changing equipment such as compressors and expanders. The placement of compressors and expanders refers to the inlet temperature of these units. The main reason for this added complexity is the fact that the shape of the GCC will change. A related complicating factor is the fact that location of pinch points may change. In addition, both heat and work are involved when compressors and expanders are integrated with heat recovery systems. The placement of compressors was briefly discussed in previous work with focus on reactor systems.[4,9] More recently, the appropriate placement of compressors and expanders was discussed by Aspelund et al.[10] and formulated as heuristic rules:

- compression adds heat to the system and should preferably be done above pinch;
- expansion provides cooling to the system and should preferably be done below pinch.

Gundersen et al.[11] stated these rules more specifically in the sense that both compression and expansion should start at the pinch temperature. Utilizing this new insight, a recuperative vapor recompression cryogenic air distillation process was developed by Fu and Gundersen.[12] The rules are applied in an Extended Pinch Analysis and Design (ExPAnD) methodology developed by Aspelund et al.[10] Following the ExPAnD methodology, Wechsung et al.[13] presented an MINLP optimization formulation for the synthesis of HENs at subambient conditions including compression and expansion. The work is further extended by Onishi et al.[14] using a superstructure with the objective of minimizing total annualized cost. Fu et al.[15] presented an MINLP optimization study for using the heat from

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Correspondence concerning this article should be addressed to C. Fu at chao.fu@ntnu.no.

air compression processes to preheat boiler feedwater in regenerative steam Rankine cycles.

In contrast to HENs, the work exchange networks focus on the matches of compressors and expanders so that pressure manipulations can be performed directly without power consumed or produced, for example, compressors and expanders can be located on a single shaft. Razib et al.[16] investigated this topic using an MINLP model. The operating temperatures of compressors and expanders are manipulated by utility exchangers. The heat exchange between process streams is not included in their study. Liu et al.[17] investigated work exchange networks using a graphical method. The inlet temperatures of compressors and expanders are fixed. Heat exchange is not included in the study. Onishi et al.[18] presented a comprehensive superstructure for work exchange networks that interact with HENs. The streams may be heated or cooled before and after each pressure manipulation stage. Stream splitting was included in each pressure manipulation stage, however, heating or cooling of stream branches within a stage was excluded. The superstructure was further extended by the same authors[19] for retrofit applications of HENs with pressure recovery of process streams. Dong et al.[20] presented an optimization study on heat, mass, and pressure exchange networks based on exergoeconomic analysis.

The integration of compressors and expanders into HENs following the heuristic rules presented by Aspelund et al.[10] is not a straightforward task. Analytical analysis and quantitative assessment of the rules based on thermodynamics and mathematics have not been performed. Direct application of the rules may even increase exergy consumption (see Example 6 in this article). Several challenges exist when applying the rules to HEN design[10,11,13,14,18,19]: (1) both heat and work are involved; (2) the streams to be compressed or expanded are included in the HEN, thus, the heating and cooling demands for the streams are changing; (3) the role of a stream (as hot or cold) may change with pressure manipulations; and (4) the heating or cooling of a stream resulting from pressure manipulation may change the location of pinch points. The splitting of streams with pressure manipulations may result in significant energy savings, however, this complex topic, while handled in this article, is not included in the models presented in the literature.[10,13,14]

The above literature survey clearly shows increasing research interest in the integration of heat and work, however, investigations have only to a limited extent been performed based on thermodynamic insights and mathematical analysis. This article, with a focus on the integration of expanders into HENs above ambient temperature, presents a systematic graphical methodology for HEN design including expanders. Since both heat and work are involved, the objective is to minimize exergy consumption (or maximize exergy production). Four theorems are proposed and proven with certain well-defined conditions. The contributions of this article include: (1) the correct integration of expanders in above ambient HENs has been investigated based on thermodynamic and mathematical analyses; (2) a graphic design procedure for above ambient HEN design including expanders has been developed; and (3) exergy analysis has been used as a predesign tool, that is, minimum exergy consumption has been achieved at an early stage of process design. The results provide useful insights and guidelines for further mathematical optimization studies. The work presented in this article has significant application potentials for improving energy efficiency and decreasing energy consumption and emissions in industrial processes where the conversion between heat and work is involved.

## Problem Statement

The problem to be solved is stated in the following way: "Given a set of process streams with supply and target states (temperature and pressure), as well as utilities for power, heating and cooling, design a network of heat exchangers, compressors, expanders, and valves in such a way that exergy consumption is minimized." For this article, it is assumed that only one stream with expansion is included and the inclusion of multiple streams to be compressed and expanded with multiple stages will be presented in other publications due to space limitations. The following assumptions are made for deriving Theorems 1–4 with the objective of minimizing exergy consumption: (1) supply and target states (temperature and pressure) for process streams and utilities for heating and cooling are given; (2) only one stream is expanded and only one hot utility (one temperature level) is used; (3) the expander polytropic efficiency $\eta_{\infty,\text{comp}}$ is constant, (4) the fluid to be expanded is ideal gas with a constant specific heat ratio of $\kappa \equiv c_p/c_v$, and (5) the exergy content of cold utility is negligible.

## Theorems

Four theorems are proposed in this section. Subsequently, these theorems are used as the basis of a design methodology.

**Theorem 1.** *For above ambient processes, a HEN design with Pinch Expansion (expansion starts at the pinch temperature) consumes the smallest amount of exergy if the following conditions are satisfied: (1) the outlet temperature of expansion at the hot utility temperature is higher than ambient temperature, and (2) Pinch Expansion does not remove the original pinch point.*

Theorem 1 can be proven in the following way. A cold stream is assumed to be expanded from $p_s$ to $p_t$. In the case that a hot stream is expanded, a similar proof can be established. More work is recovered when expansion is implemented at a higher temperature, thus, expansion at a temperature below pinch produces less work (and less cooling) compared to Pinch Expansion. As long as the hot utility demand does not increase, Pinch Expansion is preferred rather than expansion below pinch temperature to recover more work. Even more work is produced for expansion above pinch temperature, however; in that case the hot utility demand increases since the cold stream needs additional heating above pinch. Since both thermal energy (utility heat required) and mechanical energy (work produced) change, exergy consumption has been chosen for the comparison between Pinch Expansion and expansion above pinch temperature. The composite curves (CCs) for a HEN before including pressure manipulation are shown in Figure 1. The heat demand is $Q_{\text{HU},0}$ and the pinch temperature is $T_{\text{PI}}$ for cold streams. Two cases (illustrated in Figure 1) are compared: Case A, expansion starts above pinch temperature, $T_{\text{A}} > T_{\text{PI}}$; Case B, Pinch Expansion is used and, thus, $T_{\text{B}} = T_{\text{PI}}$.

For Case A, the expansion work is $W_{\text{A}} = mc_p (T_{\text{A}} - T_{\text{exp,A}}) = mc_p T_{\text{A}}[1-(p_t/p_s)^{(n_e-1)/n_e}]$, where $(n_e-1)/n_e = \eta_{\infty,\text{exp}}(\kappa-1)/\kappa$, and $\eta_{\infty,\text{exp}}$ is the expander polytropic efficiency. Since the cold stream temperature is reduced from $T_{\text{A}}$ to $T_{\text{exp,A}}$ after expansion, the heat demand increases by an amount equal to the work produced by expansion above pinch temperature, unless the original pinch point is removed and the heat demand increases even further. The heat demand for Case A is $Q_{\text{A}} = Q_{\text{HU},0} + xW_{\text{A}}$ where $x$ is the fraction expanded above pinch, $0 < x \leq 1$. The exergy consumption is

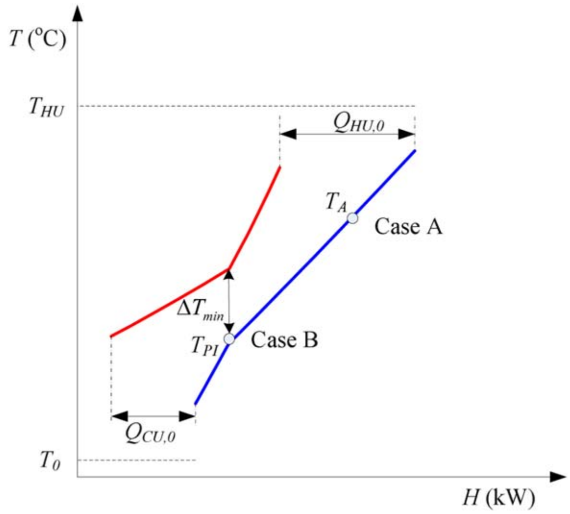


**Figure 1. CCs without pressure manipulation.**

**[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]**

$E_A = E_{Q_A} - W_A = (Q_{HU,0} + xW_A)(1 - T_0/T_{HU}) - W_A$. Similarly for Case B, the work is $W_B = mc_p(T_B - T_{exp,B}) = mc_p T_{PI}[1 - (p_t/p_s)^{(n_e-1)/n_e}]$, the heat demand is $Q_B = Q_{HU,0}$ (since the hot utility demand does not increase according to condition (2), that is, the original pinch point is not removed), and the exergy consumption is $E_B = E_{Q_B} - W_B = Q_{HU,0}(1 - T_0/T_{HU}) - W_B$. The exergy consumption of the two cases can, thus, be compared.

When $x=1$, that is, $T_{exp,A} \geq T_{PI}$, $Q_A = Q_{HU,0} + W_A = Q_{HU,0} + mc_p(T_A - T_{exp,A})$. The difference in exergy consumption for the two cases is derived to be $E_B - E_A = W_A T_0/T_{HU} - W_B = mc_p[1 - (p_t/p_s)^{(n_e-1)/n_e}](T_A T_0/T_{HU} - T_{PI})$. If there are process streams with temperature above $T_{HU}$, these streams are in heat balance (heat "pocket"). Introducing expansion in this region would introduce heat deficit and the need for another hot utility, however; only one hot utility ($T_{HU}$) is assumed, thus, $T_A \leq T_{HU}$. In addition, the scope of the article is to study integration of expanders above ambient temperature, thus, $T_{PI} > T_0$. As a result, the following inequality is satisfied: $T_A T_0/T_{HU} \leq T_0 < T_{PI}$. In the case of expansion, $p_t < p_s$, and as a result $1 - (p_t/p_s)^{(n_e-1)/n_e} > 0$ since the exponent is positive. Based on this, it is proven that $E_B - E_A < 0$, that is, Case B (Pinch Expansion) consumes a smaller amount of exergy.

When $0 < x < 1$, that is, $T_{exp,A} < T_{PI}$, $Q_A = Q_{HU,0} + xW_A = Q_{HU,0} + mc_p(T_A - T_{PI})$. The difference in exergy consumption can then be found

$$\begin{aligned} E_B - E_A &= W_A - xW_A(1 - T_0/T_{HU}) - W_B \\ &= mc_p(T_A - T_{exp,A}) - mc_p(T_A - T_{PI})(1 - T_0/T_{HU}) - mc_p(T_{PI} - T_{exp,PI}) \\ &= mc_p(T_A - T_{PI})[T_0/T_{HU} - (p_t/p_s)^{(n_e-1)/n_e}] \\ &= mc_p(T_A - T_{PI})(T_0 - T_{exp,HU})/T_{HU} \end{aligned}$$

Since $T_{exp,HU} > T_0$ according to condition (1) and $T_A > T_{PI}$, it is proven that $E_B - E_A < 0$, and again Case B (i.e., Pinch Expansion) consumes a smaller amount of exergy. In conclusion, Theorem 1 has been proven. The cases where the two assumptions in Theorem 1 are not valid (i.e., the original pinch point is removed by Pinch Expansion or $T_{exp,HU} \leq T_0$), will be discussed in Theorems 2, 3, and 4, respectively.

The hot utility demand will increase if Pinch Expansion removes the original pinch point, which should be avoided according to condition (2) in Theorem 1. Figure 2 shows the Grand Composite Curve (GCC) without pressure manipulation. Modified temperatures ($T'$) are used, which means that for cold streams $T' = T + 0.5\Delta T_{min}$, and for hot streams $T' = T - 0.5\Delta T_{min}$. The outlet temperature of Pinch Expansion is $T'_{exp,PI}$. A temperature is defined as a Potential Pinch Point if it may create a new pinch point after a portion of the cooling effect of Pinch Expansion is included (i.e., the heat surplus below pinch is reduced). The following temperatures are Potential Pinch Points: (1) the convex kink points (see definition below) on the GCC in the region between $T = T'_{PI}$ and $T = T'_{exp,PI}$ (such as points $b$ and $c$); (2) the point $T = T'_{exp,PI}$ on the GCC (point $e$) or the point (not shown in Figure 2) with $H = Q_{CU,0}$ on the line $T = T'_{exp,PI}$ if $T'_{exp,PI}$ is lower than the lowest temperature on the GCC (point $f$); (3) the potential intersection point between the constant temperature line $T = T'_{exp,PI}$ and a heat pocket (point $g$) in the GCC. A convex kink point on the GCC is defined as a point where either the slope decreases without sign change or the slope increases with sign change when referring to the positive $y$ axis direction (i.e., modified temperature).

The maximum fraction of the stream that can be expanded at the pinch, $(mc_p)_{exp,PI,max}$, is determined by the following steps: (1) starting at the pinch point ($T'_{PI}$), draw lines between the pinch point and the Potential Pinch Points and extend the line with the largest negative slope until it intersects with the constant temperature line ($T = T'_{exp,PI}$). The corresponding cooling demand $Q_{exp,max}$ at the intersection (point $h$ in Figure 2) is then determined; (2) this cooling demand is equal to the maximum work resulting from Pinch Expansion, and $(mc_p)_{exp,PI,max}$ can, thus, be determined as $(mc_p)_{exp,PI,max} = Q_{exp,max}/(T'_{PI} - T'_{exp,PI})$. If the $mc_p$ of the stream to be expanded is larger than $(mc_p)_{exp,PI,max}$, the original pinch point will be removed and the hot utility demand will increase if the entire stream is expanded. Stream splitting is used to avoid

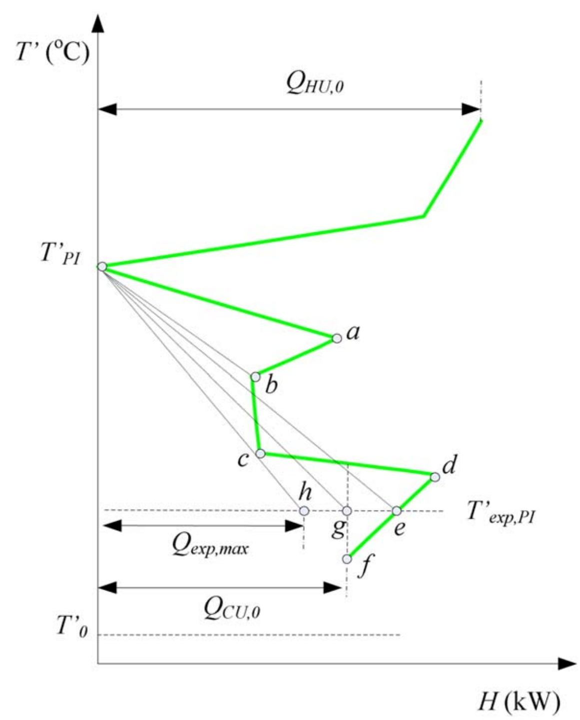


**Figure 2. GCC without pressure manipulation.**

**[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]**

**Table 1. Stream Data for Example 1**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| H1 | 400 | 60 | 3 | 1020 | – | – |
| C1 | 300 | 380 | 2 | 160 | 300 | 100 |
| C2 | 200 | 380 | 6 | 1080 | – | – |
| Heat source | 400 | 400 | – | – | – | – |
| Cooling source | 15 | 15 | – | – | – | – |

this and the fraction of the stream using Pinch Expansion is $(mc_p)_{\mathrm{exp,PI,max}}$.

The procedure above to identify the maximum fraction of Pinch Expansion and the discussion about Potential Pinch Points did neither reveal the type (hot or cold) nor the supply and target temperatures of the streams. A detailed study on the effect of stream type (hot/cold) and relative position of stream temperatures (supply/target), expander outlet temperature and pinch temperature is provided in Appendix A. This study concludes that the effect of considering stream type and corresponding supply and target temperatures in cases with Pinch Expansion is an increase in hot utility demand equal to $ymc_p\Delta T_{\mathrm{min}}$, where $0 \leq y \leq 1$. This amount normally represents a negligible contribution to the total exergy consumption of the process for two reasons: (1) $\Delta T_{\mathrm{min}}$ is normally small and, thus, $ymc_p\Delta T_{\mathrm{min}}$ is small compared to the total hot utility demand, and (2) the exergy content of this heat, $ymc_p\Delta T_{\mathrm{min}}(1-T_0/T_{\mathrm{HU}})$, is even smaller. The advantage of excluding such negligible hot utility demand in the analysis is that the GCC can be used without distinguishing the type of streams to be expanded and the location of supply and target temperatures. The inherent challenge with compression and expansion that the identity of the streams may change is then no longer a problem. A comprehensive procedure can of course be developed to include all the cases presented in Appendix A when $\Delta T_{\mathrm{min}}$ is large and the additional hot utility demand ($ymc_p\Delta T_{\mathrm{min}}$) needs to be taken into account.

The following assumptions are made for Examples 1–6: (1) $\eta_{\infty,\mathrm{exp}}=1$, (2) $\Delta T_{\mathrm{min}}=20$°C, (3) $T_0=15$°C, and (4) the fluid to be expanded is ideal gas with constant specific heat ratio$\kappa=1.4$.

**Example 1**. The stream data is shown in Table 1, indicating that a cold stream (C1) is expanded. The outlet temperature of expansion at $T_{\mathrm{HU}}$ is $T_{\mathrm{exp,HU}}=T_{\mathrm{HU}}(p_t/p_s)^{(n_e-1)/n_e}=218.7°\mathrm{C}> T_0$, thus, according to Theorem 1, Pinch Expansion can reduce exergy consumption.

The GCC without pressure manipulation (Case O) is shown in Figure 3a. The pinch temperature is 220/200°C. The following four cases are compared: Case A, expansion at $T_s$ (300°C); Case B, expansion at $T_{\mathrm{PI}}$ for the cold streams (200°C); Case C, expansion at $T_{\mathrm{HU}}-\Delta T_{\mathrm{min}}$ (380°C); Case D, the outlet temperature of expansion is equal to $T_0$ (15°C). Due to expansion at different temperatures, the stream data for C1 changes as shown in Table 2. In Case A, the only change is a reduced supply temperature. The stream expansion results in two new streams in Cases B, C, and D. In addition, the role of the stream changes in Cases B and D, that is, the stream temporarily becomes a hot stream (C1_1) before expansion. The corresponding GCCs are shown in Figure 3. The original pinch is not removed in these cases, but the utility demands change when expansion is implemented.

The performance results are listed in Table 3. Note that Case O is included to show the utility demands and the pinch temperature for the case where pressure manipulation is not included. It is, thus, not reasonable to compare the exergy

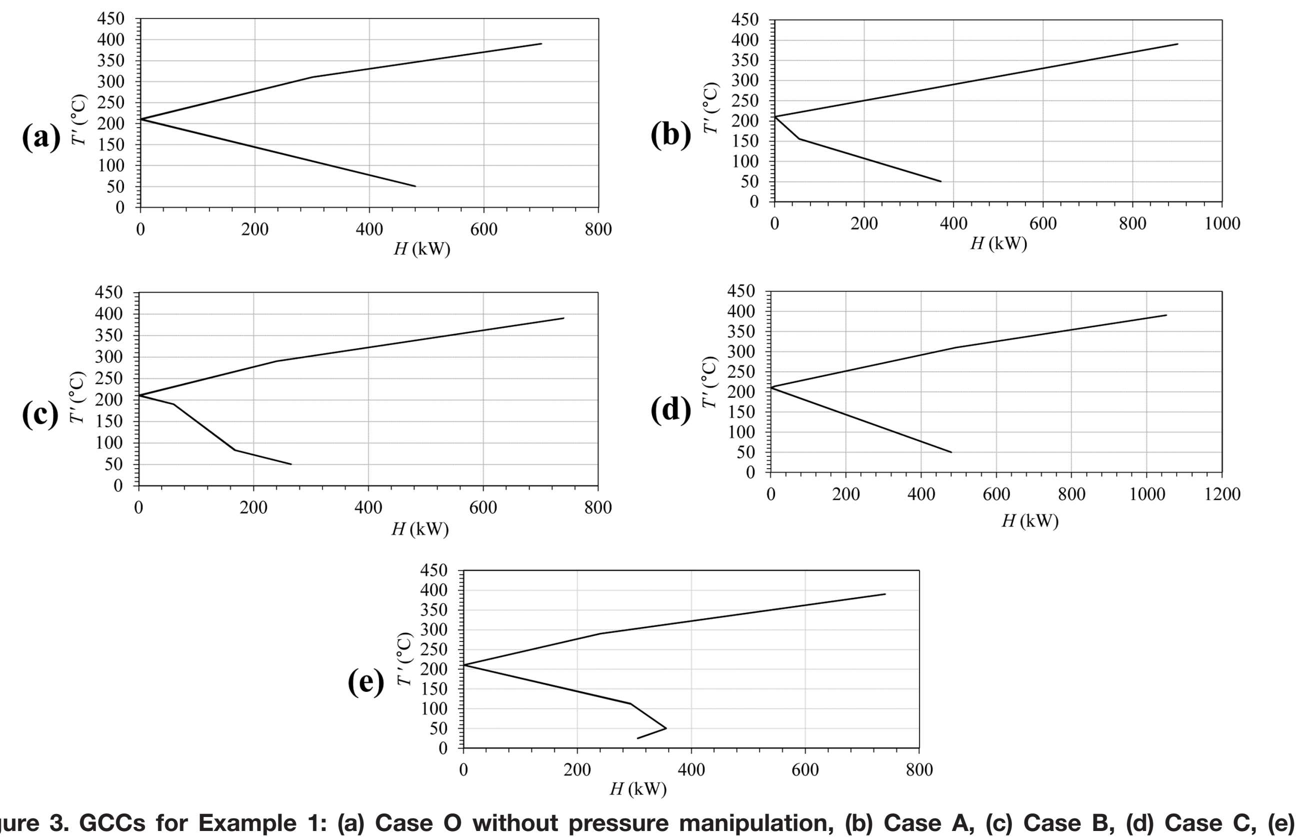


**Figure 3. GCCs for Example 1: (a) Case O without pressure manipulation, (b) Case A, (c) Case B, (d) Case C, (e) Case D.**

**Table 2. Stream Data for C1 in Example 1**

| Cases | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| **Case A** | | | | | | |
| C1 | 145.6 | 380 | 2 | 268.8 | 100 | 100 |
| **Case B** | | | | | | |
| C1_1 | 300 | 200 | 2 | 200 | 300 | 300 |
| C1_2 | 72.5 | 380 | 2 | 615 | 100 | 100 |
| **Case C** | | | | | | |
| C1_1 | 300 | 380 | 2 | 160 | 300 | 300 |
| C1_2 | 204 | 380 | 2 | 352 | 100 | 100 |
| **Case D** | | | | | | |
| C1_1 | 300 | 122.3 | 2 | 355.4 | 300 | 300 |
| C1_2 | 15 | 380 | 2 | 730 | 100 | 100 |

consumption between Case O and other cases. Expansion at a higher temperature (e.g., Case C) increases the expansion work but also the hot utility demand. Pinch Expansion consumes the smallest amount of exergy. Compared to expansion at $T_s$ (Case A), the exergy consumption is reduced by 18.3%. Notice that compared to Case O (without pressure manipulation), the hot utility for Pinch Expansion increases by 40 kW, which is exactly equal to $mc_p\Delta T_{min}$ as explained and illustrated in Appendix A. Expansion below pinch (Case D) reduces the cooling demand, however, the exergy consumption is more than Case B where Pinch Expansion is used since less work is produced by the expansion.

The HEN designs are shown in Figure 4. Cases A and C have the same number of units, while Cases B and D require 1 more heat exchanger. A detailed cost analysis (including assumptions used) for various cases is presented in Appendix B. The annualized capital cost (CAPEX), operating cost (OPEX), and total annualized cost (TAC) are included in Table 3. Pinch Expansion (Case B) has the smallest TAC among these cases. Compared to Case B, the TAC is increased by 16.6% and 31.8% for Cases C and D, respectively. Pinch Expansion is, thus, a cost effective alternative in this special example. Case A may also be cost competitive considering the uncertainties related to cost analysis.

**Theorem 2.** *For above ambient processes, if the outlet temperature of expansion at hot utility temperature is higher than the pinch temperature and the cooling produced by Pinch Expansion is more than required, the stream to be expanded should be split into two portions: the first portion (as large as possible until the cooling demand is satisfied) is expanded at the pinch temperature and the remaining portion is expanded at either hot utility temperature or ambient temperature.*

Theorem 2 is proven in the following way. Consider the case when a cold stream is expanded from $p_s$ to $p_t$. In the case that a hot stream is expanded, a similar proof can be established. Since the cooling demand can be completely satisfied by Pinch Expansion, the outlet temperature of Pinch Expansion, $T_{exp,PI}$, should be lower than the highest possible cold utility temperature, $T_{CU,max}$. In the proof of Theorem 1, it was explained that Pinch Expansion is more favorable than below pinch expansion schemes. The comparison is, thus, performed between Pinch Expansion and expansion above pinch.

According to Theorem 1, Pinch Expansion is the preferred scheme and (portion $\alpha$) should be maximized. When the cooling demand is satisfied, the remaining expansion (portion $\gamma$) will result in increased hot utility requirements independent of expansion inlet temperature, unless expansion starts at $T_0$ as will be discussed later. It is important to notice that expansion above pinch will increase hot utility consumption by the same amount as the work produced in expansion. Thus, heat is completely converted into work, which of course is thermodynamically favorable. Expansion at higher temperature is, thus, preferred, since it increases the amount of heat that can be converted into work on a 1:1 basis, and the ultimate situation would be to expand at $T_{HU}$.

Another alternative is expansion at $T_0$ (Ambient Expansion). After being cooled to $T_0$, the stream is expanded to subambient temperature ($T_{exp,0}$), and then heated to $T_0$ by cold utility, before finally being heated to $T_s$ by the heat available from cooling the stream to $T_0$ (recuperative heating). This alternative does not increase the hot utility demand.

The following two cases are compared for expansion of the remaining portion $\gamma$ after Pinch Expansion has been fully utilized (the cooling demand has been satisfied): Case A, expansion at $T_{HU}$, and Case B, expansion at $T_0$. For Case A, the expansion work is $W_A=(mc_p)_\gamma T_{HU}[1-(p_t/p_s)^{(n_e-1)/n_e}]$, the heat demand is $Q_A=Q_{HU,0}+W_A$ (since Theorem 2 considers the case when $T_{PI} < T_{exp,HU}$), and the exergy consumption is $E_A=E_{Q_A}-W_A$. Similarly for Case B, the work is $W_B=(mc_p)_\gamma T_0[1-(p_t/p_s)^{(n_e-1)/n_e}]$, the heat demand is $Q_B=Q_{HU,0}$, and the exergy consumption is $E_B=E_{Q_B}-W_B$. The exergy consumption in the two cases can, thus, be compared

$$\begin{aligned} E_A-E_B &= (Q_{HU,0}+W_A)(1-T_0/T_{HU})-W_A \\ &\quad -Q_{HU,0}(1-T_0/T_{HU})+W_B \\ &= W_B-(T_0/T_{HU})W_A \\ &= (mc_p)_\gamma[1-(p_t/p_s)^{(n_e-1)/n_e}][T_0-(T_0/T_{HU})T_{HU}] \\ &= 0 \end{aligned}$$

This proves that the exergy consumption in the two cases is the same. Thus, once the cooling demand has been satisfied, the remaining portion of the stream can be expanded at either $T_{HU}$ or $T_0$. Note that when Ambient Expansion is used,

**Table 3. Performance Comparison for Example 1**

| Cases | O | A | B | C | D |
|---|---|---|---|---|---|
| Hot utility demand (kW) | 700 | 900 | 740 | 1052 | 740 |
| Cold utility demand (kW) | 480 | 371.2 | 265 | 480 | 305.4 |
| Pinch temperature (°C) | 210 | 210 | 210 | 210 | 210 |
| Expansion work (kW) | – | 308.8 | 255 | 351.9 | 214.6 |
| Exergy consumption (kW) | – | 205.9 | 168.2 | 249.8 | 208.6 |
| Number of heat exchangers | – | 5 | 6 | 5 | 6 |
| Number of expanders | – | 1 | 1 | 1 | 1 |
| CAPEX (kUSD) | – | 36.145 | 39.237 | 35.944 | 41.107 |
| OPEX (kUSD) | – | 23.45 | 18.52 | 31.4 | 35.003 |
| TAC (kUSD) | – | 59.595 | 57.757 | 67.344 | 76.11 |

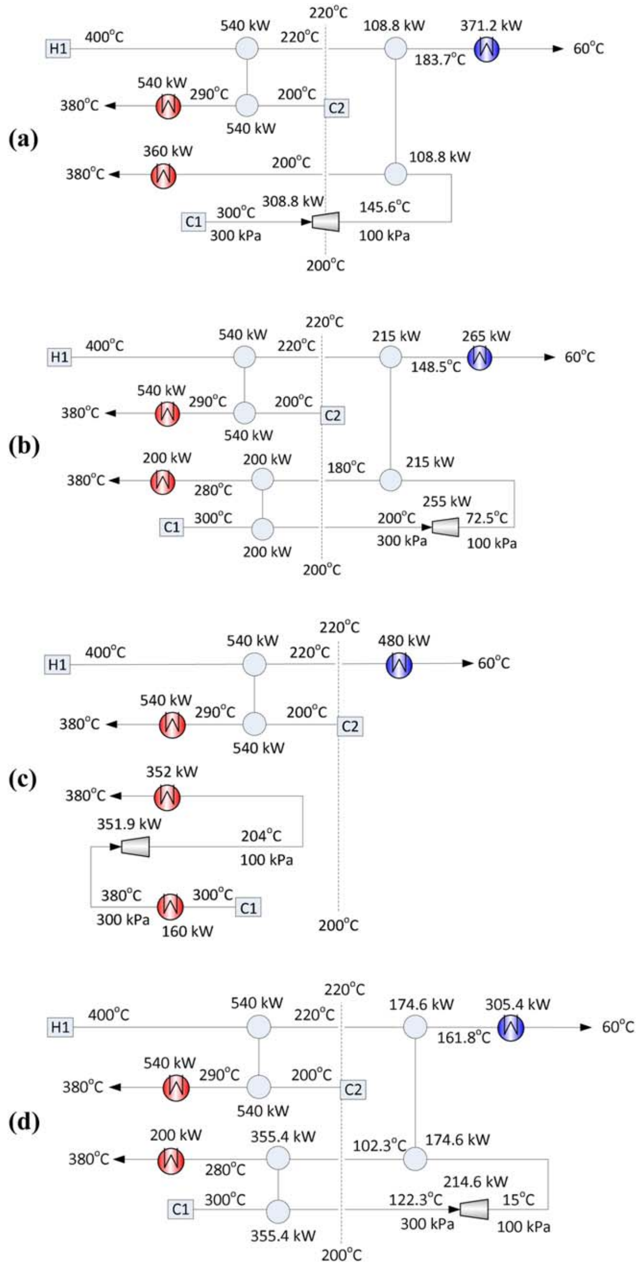


**Figure 4. Heat exchanger networks for Example 1: (a) Case A, (b) Case B, (c) Case C, (d) Case D.**

**[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]**

refrigeration energy is produced and its exergy has not been included in the comparison.

This result is somewhat surprising since it was argued that beyond Pinch Expansion that completely covers the cooling demand, the remaining expansion should be done at the highest possible temperature, since this maximizes the work produced by heat on a 1:1 basis. The explanation is that Ambient Expansion does not cause additional hot utility consumption. The difference in work (exergy) produced by expansion matches exactly the exergy of the additional hot utility consumption when expansion at $T_{\rm HU}$ is used. This can be seen by rearranging the exergy expression used to prove $E_{\rm A}=E_{\rm B}$: $E_{\rm A}-E_{\rm B}=W_{\rm A}(1-T_0/T_{\rm HU})-(W_{\rm A}-W_{\rm B})=0$, where the first term is the exergy of the additional amount of hot utility for Case A, while the second term $(W_{\rm A}-W_{\rm B})$ is the additional work (exergy) produced in Case A compared to Case B. As shown, these two terms balance and the surprising result is physically explained.

The next comparison is between Pinch Expansion (with stream splitting) and expansion at some intermediate temperature $T_{\rm C}(T_{\rm PI}<T_{\rm C}<T_{\rm HU})$. The following two cases are compared: Case A, the stream $(mc_p)$ to be expanded is split into two portions: one portion $(\alpha)$ is expanded at $T_{\rm PI}$ and the work is equal to the cooling demand $Q_{\rm CU,0}$, and the remaining portion $(\gamma)$ is expanded at $T_{\rm HU}$; Case C, expansion starts at temperature $T_{\rm C}$ and the cooling demand is satisfied by the expansion. Case C actually includes two subcases: the first case (C.i) is when the cooling demand is just satisfied and $T_{\rm exp,C}\le T_{\rm CU,max}$ (the stream is not split); the second case (C.ii) is when the cooling introduced by expansion is more than required and the stream is split: the first portion is expanded at $T_{\rm C}$ so that the cooling demand is just satisfied, and the remaining portion is expanded at $T_{\rm HU}$. Note that similar to the comparison between Cases A and B, expansion of the portion $\gamma$ at $T_0$ results in the same amount of exergy consumption as expansion at $T_{\rm HU}$, and this case is, thus, not included in the comparison.

The first comparison is performed between Cases A and C.i, as is shown in Figure 5. For Case C.i, assuming that the expansion ratio below pinch is $p_{r,\rm C1}$ and above pinch is $p_{r,\rm C2}$, obviously $p_{r,\rm C1}\times p_{r,\rm C2}=p_r=p_{\rm t}/p_{\rm s}$. The work production for Case C (= C.i) is $W_{\rm C}=mc_p(T_{\rm C}-T_{\rm PI})+mc_p(T_{\rm PI}-T_{\rm exp,C})=mc_p(T_{\rm C}-T_{\rm PI})+Q_{\rm CU,0}$. The heat demand is $Q_{\rm C}=Q_{\rm HU,0}+mc_p(T_{\rm C}-T_{\rm PI})$. The exergy consumption is $E_{\rm C}=Q_{\rm C}(1-T_0/T_{\rm HU})-W_{\rm C}$. For Case A, $W_{\rm A}=(mc_p)_\alpha(T_{\rm PI}-T_{\rm exp,PI})+(mc_p)_\gamma(T_{\rm HU}-T_{\rm exp,HU})=Q_{\rm CU,0}+(mc_p)_\gamma(T_{\rm HU}-T_{\rm exp,HU})$, $Q_{\rm A}=Q_{\rm HU,0}+(mc_p)_\gamma(T_{\rm HU}-T_{\rm exp,HU})$, and $E_{\rm A}=Q_{\rm A}(1-T_0/T_{\rm HU})-W_{\rm A}$.

Since the cooling demand is just satisfied in both cases, $Q_{\rm CU,0}=(mc_p)_\alpha(T_{\rm PI}-T_{\rm exp,PI})=mc_p(T_{\rm PI}-T_{\rm exp,C})$. The following two equations can then be obtained

$$\frac{(mc_p)_\alpha}{mc_p}=\frac{T_{\rm PI}-T_{\rm exp,C}}{T_{\rm PI}-T_{\rm exp,PI}}=\frac{T_{\rm PI}[1-p_{r,\rm C1}{}^{(n_{\rm e}-1)/n_{\rm e}}]}{T_{\rm PI}[1-p_r{}^{(n_{\rm e}-1)/n_{\rm e}}]}=\frac{1-p_{r,\rm C1}{}^{(n_{\rm e}-1)/n_{\rm e}}}{1-p_r{}^{(n_{\rm e}-1)/n_{\rm e}}}$$

$$\frac{(mc_p)_\gamma}{mc_p}=\frac{mc_p-(mc_p)_\alpha}{mc_p}=\frac{p_{r,\rm C1}{}^{(n_{\rm e}-1)/n_{\rm e}}-p_r{}^{(n_{\rm e}-1)/n_{\rm e}}}{1-p_r{}^{(n_{\rm e}-1)/n_{\rm e}}}$$

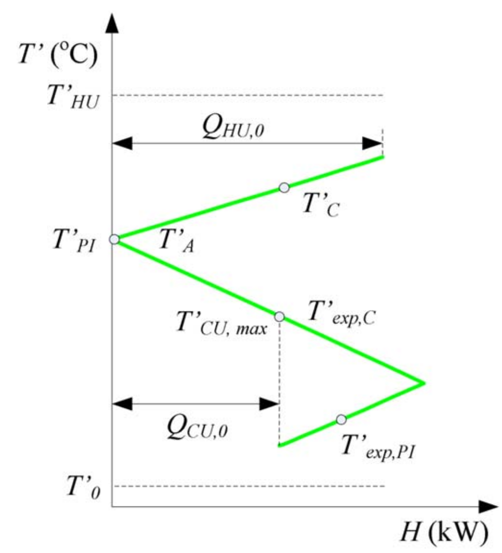


**Figure 5. GCC for Theorem 2.**

**[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]**

**Table 4. Stream Data for Example 2**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| H1 | 400 | 110 | 2 | 580 | – | – |
| H2 | 400 | 280 | 3 | 360 | 250 | 100 |
| C1 | 200 | 380 | 8 | 1440 | – | – |
| Heat source | 400 | 400 | – | – | – | – |
| Cooling source | 15 | 15 | – | – | – | – |

The exergy consumption for the two cases can then be compared

$$
\begin{aligned}
& E_A - E_C \\
&= (Q_A - Q_C)(1 - T_0/T_{HU}) - (W_A - W_C) \\
&= mc_p (T_0/T_{HU}) \left[ (T_C - T_{PI}) - \frac{(mc_p)_\gamma}{mc_p} (T_{HU} - T_{exp,HU}) \right] \\
&= mc_p (T_0/T_{HU}) \left\{ T_{PI} \frac{[1 - p_{r,C2}{}^{(n_e-1)/n_e}]}{p_{r,C2}{}^{(n_e-1)/n_e}} \right. \\
& \left. - \frac{[p_{r,C1}{}^{(n_e-1)/n_e} - p_r{}^{(n_e-1)/n_e}]}{[1 - p_r{}^{(n_e-1)/n_e}]} T_{HU} [1 - p_r{}^{(n_e-1)/n_e}] \right\} \\
&= \frac{mc_p T_0 [1 - p_{r,C2}{}^{(n_e-1)/n_e}]}{T_{HU} p_{r,C2}{}^{(n_e-1)/n_e}} [T_{PI} - T_{HU} p_r{}^{(n_e-1)/n_e}]
\end{aligned}
$$

Since $1 - p_{r,C2}{}^{(n_e-1)/n_e} > 0$ and $T_{PI} < T_{exp,HU} = T_{HU} p_r{}^{(n_e-1)/n_e}$ (according to the condition in Theorem 2), thus, $E_A - E_C < 0$ and Case A consumes less exergy.

For Case C.ii, the stream is split and one portion is expanded at $T_{HU}$. This portion can be removed from the comparison by subtracting an equal portion with expansion at $T_{HU}$ in Case A. The previous proof is then applied and the same conclusion can be achieved. There may also be cases where the stream is split into many portions that are expanded at different temperatures above pinch. Obviously the expansion of each portion results in more exergy consumption compared to the case where Pinch Expansion combined with expansion at $T_{HU}$ (if necessary) are used for the same portion. The total effect is that Case A has the minimum exergy consumption. Theorem 2 has, thus, been proven.

Example 2. The stream data is shown in Table 4, indicating that a hot stream (H2) is expanded. The outlet temperature of expansion at $T_{HU}$ is $T_{exp,HU} = T_{HU}(p_t/p_s)^{(n_e-1)/n_e} = 245.0°C > T_0$, thus, Pinch Expansion can reduce the exergy consumption according to Theorem 1. The GCC without pressure manipulation (Case O) is shown in Figure 6a. The pinch temperature is 220/200°C. Since $T_{exp,HU} > T_{PI}$, Theorem 2 is applicable. The outlet temperature of expansion at $T_{PI}$ is calculated to be $T_{exp,PI} = 106.4°C$.

The following two cases are compared: Case A, Pinch Expansion is used for a maximum portion of the stream and the remaining portion is expanded at $T_{HU}$; Case B, Pinch Expansion is used for the same portion as in Case A and the remaining portion is expanded at $T_0$. According to Figure 6a, the maximum portion of the stream that can be expanded at $T_{PI}$ is calculated to be $(mc_p)_{exp,PI,max} = Q_{exp,max}/(T'_{PI} - T'_{exp,PI}) = 220/(210 - 96.4)$ kW/°C$=1.94$ kW/°C. Stream H2 with $mc_p = 3$ kW/°C, thus, needs to be split: the first portion ($\alpha = 1.94$ kW/°C) is expanded after being cooled from $T_s$ to $T_{PI}$, and is then heated from $T_{exp,PI}$ to $T_t$. The remaining portion ($\gamma = 1.06$ kW/°C) is either expanded directly at $T_s = T_{HU}$ (Case A) or expanded after being cooled to $T_0$ (Case B). Of course, it is a coincidence that $T_s$ for H2 is equal to $T_{HU}$. New stream data for H2 is shown in Table 5. For the portion ($\alpha$) with Pinch Expansion, the stream becomes a cold stream after expansion. In Case B, the portion ($\gamma$) after expansion at $T_0$ is heated to $T_0 - \Delta T_{min} = -5°C$ by cold utility (in this case acting as a hot utility) and then further heated to $T_t - \Delta T_{min}$ by recuperative heating. Notice that $T_s$ is not used here since H2 is a hot stream. The GCCs are shown in Figure 6. With maximum Pinch Expansion, one would expect cooling demand in the process to be completely eliminated. However, due to the influence of $\Delta T_{min}$, as discussed in Appendix A, there will be some cooling demand left (38.4 kW) for both Cases A and B as indicated in the GCCs of Figures 6b, c and quantified in Table 6. Note that the exergy consumption for Case O is excluded since the required expansion is not implemented.

As indicated in Table 6, the exergy consumption for Cases A and B are almost the same. The reason for the slight difference is that recuperative heating of the portion $\gamma$ of stream H2 in Case B increases the hot utility demand by $(mc_p)\Delta T_{min} = 1.06 \times 20 = 21.2$ kW (for heating from $T_t - \Delta$

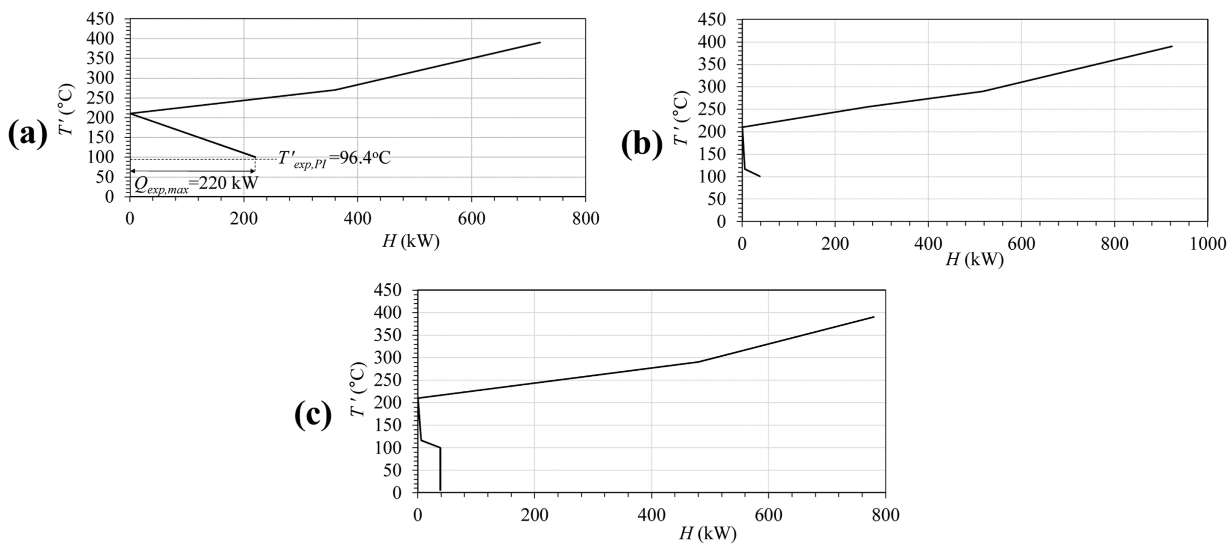


**Figure 6. GCCs for Example 2: (a) Case O without pressure manipulation, (b) Case A, (c) Case B.**

**Table 5. Stream Data for H2 in Example 2**

| Cases | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| **Case A** | | | | | | |
| H2_α1 | 400 | 220 | 1.94 | 349.2 | 250 | 250 |
| H2_α2 | 106.4 | 280 | 1.94 | 336.8 | 100 | 100 |
| H2_γ | 245.0 | 280 | 1.06 | 37.1 | 100 | 100 |
| **Case B** | | | | | | |
| H2_α1 | 400 | 220 | 1.94 | 349.2 | 250 | 250 |
| H2_α2 | 106.4 | 280 | 1.94 | 336.8 | 100 | 100 |
| H2_γ1 | 400 | 15 | 1.06 | 408.1 | 250 | 250 |
| H2_γ2 | −5 | 280 | 1.06 | 302.1 | 100 | 100 |

$T_{\min}$ to $T_t$). The corresponding exergy of this heat is 12.1 kW, which is equal to the difference (12.0 kW) between the exergy consumption in the two cases. Notice that the use of cold utility to heat the expanded stream from $T_{\exp,0}$ to $T_0-\Delta T_{\min}$ has not been included in the cold utility demand in Table 6. Actually this could be seen as generating refrigeration, which is an additional benefit in Case B.

While the focus of this article is to study the placement of expanders in HENs using exergy as the measure of design quality since both heat and work are involved, a brief discussion about the investment cost implications could be given instead of detailed cost analysis for this example. As shown in Table 6, the exergy consumption is almost equal for the two cases (A and B), while hot utility demand and work production differ significantly. Expansion at $T_0$ produces less work since the gas volumes are smaller compared to expansion at $T_{\mathrm{HU}}$, and this reduces the investment cost for the expander. The recuperative heating required in this case, however, increases the total amount of heat transfer, thus, the investment cost related to heat exchange will be larger (more units and more heat transfer area) than in the case with expansion at $T_{\mathrm{HU}}$. In addition, since the relative prices of heat and power do not always follow the second Law of Thermodynamics, there may be cases where the total energy cost of Case B is less than for Case A and vice versa.

**Theorem 3.** *For above ambient processes, if the outlet temperature of expansion at hot utility temperature is lower than the pinch temperature but higher than ambient temperature, and the cooling produced by Pinch Expansion is more than required, the cooling resulting from expansion at hot utility temperature should be utilized to reduce the portion with Pinch Expansion.*

In the case that $T_{\exp,\mathrm{HU}} < T_{\mathrm{PI}}$, expansion at $T_{\mathrm{HU}}$ introduces cooling to the region below $T_{\mathrm{PI}}$. If the original cooling demand ($Q_{\mathrm{CU},0}$) has been satisfied by Pinch Expansion, as was discussed for Theorem 2, expansion of the remaining portion of the stream increases hot utility consumption by the same amount as the work produced. Expansion at $T_{\mathrm{HU}}$ is, thus, preferred, and the portion for Pinch Expansion should then be reduced. The GCC is illustrated in Figure 7. The solid lines

**Table 6. Performance Comparison for Example 2**

| Cases | O | A | B |
|---|---|---|---|
| Hot utility demand (kW) | 720 | 923.1 | 780 |
| Cold utility demand (kW) | 220 | 38.4 | 38.4 |
| Pinch temperature (°C) | 210 | 210 | 210 |
| Expansion work (kW) | – | 384.7 | 324.6 |
| Exergy consumption (kW) | – | 143.3 | 155.3 |

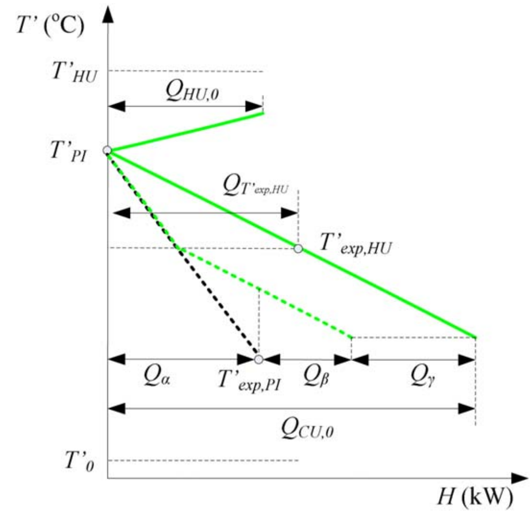


**Figure 7. GCC for Theorem 3.**

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

represent the GCC without pressure manipulation. Due to expansion at $T'_{\mathrm{HU}}$, the amount of cooling introduced to the region below pinch is $Q_\gamma$. The dashed lines (in green) represent the new GCC below $T'_{\mathrm{PI}}$ after pressure manipulation of the portion expanded at $T'_{\mathrm{HU}}$ has been included. The portion of cooling provided by Pinch Expansion (stream fraction α) is reduced from $Q_{\mathrm{CU},0}$ to $Q_\alpha$. A portion (referred to as β) should be expanded at $T'_{\exp,\mathrm{HU}}$ if this is a new pinch temperature. The following two cases exist and are discussed

1. $mc_p(T'_{\mathrm{PI}}-T'_{\exp,\mathrm{HU}}) < Q_{T'_{\exp,\mathrm{HU}}}$

Here, $Q_{T'_{\exp,\mathrm{HU}}}$ is the cooling demand at $T'_{\exp,\mathrm{HU}}$. No new pinch is created at $T'_{\exp,\mathrm{HU}}$. The portion β is, thus, not required since expansion below pinch should be avoided. The portions α and γ are then determined by the following equations

$$(mc_p)_\alpha(T'_{\mathrm{PI}}-T'_{\exp,\mathrm{PI}})+(mc_p)_\gamma(T'_{\mathrm{PI}}-T'_{\exp,\mathrm{HU}})=Q_{\mathrm{CU},0}$$

$$(mc_p)_\alpha+(mc_p)_\gamma=mc_p$$

The scheme proposed above (referred to as Case B) can be compared with expansion above pinch (Case A) in the following way. For Case A, assume that expansion of the entire stream starts at $T_{\mathrm{A}}$ ($T_{\mathrm{PI}} < T_{\mathrm{A}} < T_{\mathrm{HU}}$) and that cooling demand is satisfied by this expansion. The pressure ratios below and above pinch are $p_{r,\mathrm{A}1}$ and $p_{r,\mathrm{A}2}$, respectively. The following relations can be established: $p_{r,\mathrm{A}1}\times p_{r,\mathrm{A}2}=p_r=p_t/p_s$, $T_{\mathrm{PI}}=T_{\mathrm{A}}p_{r,\mathrm{A}2}{}^{(n_e-1)/n_e}$, and $T_{\exp,\mathrm{A}}=T_{\mathrm{A}}p_r{}^{(n_e-1)/n_e}$. The work production for Case A is $W_{\mathrm{A}}=mc_p(T_{\mathrm{A}}-T_{\exp,\mathrm{A}})$, the heat demand is $Q_{\mathrm{A}}=Q_{\mathrm{HU},0}+mc_p(T_{\mathrm{A}}-T_{\mathrm{PI}})$, and the exergy consumption is $E_{\mathrm{A}}=Q_{\mathrm{A}}(1-T_0/T_{\mathrm{HU}})-W_{\mathrm{A}}$.

For Case B (Pinch Expansion combined with expansion at $T_{\mathrm{HU}}$), assuming that the pressure ratios below and above pinch are $p_{r,\mathrm{B}1}$ and $p_{r,\mathrm{B}2}$, respectively, for the portion γ, the following relations can be established: $p_{r,\mathrm{B}1}\times p_{r,\mathrm{B}2}=p_r=p_t/p_s$, $T_{\mathrm{PI}}=T_{\mathrm{HU}}p_{r,\mathrm{B}2}{}^{(n_e-1)/n_e}$, and $T_{\exp,\mathrm{HU}}=T_{\mathrm{HU}}p_r{}^{(n_e-1)/n_e}$. The work production is $W_{\mathrm{B}}=(mc_p)_\alpha(T_{\mathrm{PI}}-T_{\exp,\mathrm{PI}})+(mc_p)_\gamma(T_{\mathrm{HU}}-T_{\exp,\mathrm{HU}})$, the heat demand is $Q_{\mathrm{B}}=Q_{\mathrm{HU},0}+(mc_p)_\gamma(T_{\mathrm{HU}}-T_{\mathrm{PI}})$, and the exergy consumption is $E_{\mathrm{B}}=Q_{\mathrm{B}}(1-T_0/T_{\mathrm{HU}})-W_{\mathrm{B}}$.

The cooling demand is exactly satisfied by the below pinch part of the expansion in both Cases A and B, thus, $Q_{\mathrm{CU},0}=mc_p(T_{\mathrm{PI}}-T_{\mathrm{exp,A}})=(mc_p)_\alpha(T_{\mathrm{PI}}-T_{\mathrm{exp,PI}})+(mc_p)_\gamma(T_{\mathrm{PI}}-T_{\mathrm{exp,HU}})$, and $[(mc_p)_\alpha+(mc_p)_\gamma]T_{\mathrm{PI}}[1-p_{r,\mathrm{A1}}{}^{(n_e-1)/n_e}]=(mc_p)_\alpha T_{\mathrm{PI}}[1-p_r{}^{(n_e-1)/n_e}]+(mc_p)_\gamma T_{\mathrm{PI}}[1-p_{r,\mathrm{B1}}{}^{(n_e-1)/n_e}]$. The following two equations can then be obtained

$$\frac{(mc_p)_\gamma}{mc_p}=\frac{p_r{}^{(n_e-1)/n_e}-p_{r,\mathrm{A1}}{}^{(n_e-1)/n_e}}{p_r{}^{(n_e-1)/n_e}-p_{r,\mathrm{B1}}{}^{(n_e-1)/n_e}}$$

$$\frac{(mc_p)_\alpha}{mc_p}=\frac{p_{r,\mathrm{A1}}{}^{(n_e-1)/n_e}-p_{r,\mathrm{B1}}{}^{(n_e-1)/n_e}}{p_r{}^{(n_e-1)/n_e}-p_{r,\mathrm{B1}}{}^{(n_e-1)/n_e}}$$

The exergy consumption can, thus, be compared in the following way

$$\begin{aligned}
&E_\mathrm{A}-E_\mathrm{B}\\
&=(Q_\mathrm{A}-Q_\mathrm{B})(1-T_0/T_{\mathrm{HU}})-(W_\mathrm{A}-W_\mathrm{B})\\
&\qquad=mc_p\left[T_{\mathrm{exp,A}}-\frac{(mc_p)_\alpha}{mc_p}T_{\mathrm{exp,PI}}\right.\\
&-\frac{(mc_p)_\gamma}{mc_p}T_{\mathrm{exp,HU}}]-mc_p[(T_\mathrm{A}-T_{\mathrm{PI}})-\left.\frac{(mc_p)_\gamma}{mc_p}(T_{\mathrm{HU}}-T_{\mathrm{PI}})\right](T_0/T_{\mathrm{HU}})\\
&=mc_pT_{\mathrm{PI}}\left[p_{r,\mathrm{A1}}{}^{(n_e-1)/n_e}-\frac{(mc_p)_\alpha}{mc_p}p_r{}^{(n_e-1)/n_e}-\frac{(mc_p)_\gamma}{mc_p}p_{r,\mathrm{B1}}{}^{(n_e-1)/n_e}\right]\\
&-mc_pT_{\mathrm{PI}}\left[\frac{1-p_{r,\mathrm{A2}}{}^{(n_e-1)/n_e}}{p_{r,\mathrm{A2}}{}^{(n_e-1)/n_e}}-\frac{(mc_p)_\gamma}{mc_p}\frac{1-p_{r,\mathrm{B2}}{}^{(n_e-1)/n_e}}{p_{r,\mathrm{B2}}{}^{(n_e-1)/n_e}}\right](T_0/T_{\mathrm{HU}})
\end{aligned}$$

By introducing the $mc_p$ ratios given above, it can easily be derived that $E_\mathrm{A}-E_\mathrm{B}=0$. The exergy consumption is, thus, actually the same for Cases A and B.

It should be noticed that expansion above pinch ($T_{\mathrm{PI}}<T_\mathrm{A}<T_{\mathrm{HU}}$) includes other cases where the cooling introduced by expansion is more than required and the stream is, thus, split: the first portion is expanded at $T_\mathrm{A}$ and the remaining portion is expanded at $T_{\mathrm{HU}}$. The cooling demand is satisfied by the expansion of the two portions. The portion with expansion at $T_{\mathrm{HU}}$ can actually be removed from the comparison by subtracting an equal portion with expansion at $T_{\mathrm{HU}}$ in Case B. The previous comparison procedure can then be applied and the same conclusion is obtained. Similar to the proof of Theorem 2, the expansion above pinch may also include cases, where the stream is split into many portions that are expanded at different temperatures above pinch. The expansion of each portion can be compared with a corresponding case where Pinch Expansion combined with expansion at $T_{\mathrm{HU}}$ (if necessary) is used for the same portion. The general result is that the two cases (A and B) have the same exergy consumption.

In conclusion, the expansion scheme proposed in Theorem 3 consumes the smallest amount of exergy. As illustrated, however, there are cases where expansion at some intermediate temperature $T_\mathrm{A}$ ($T_{\mathrm{PI}}<T_\mathrm{A}<T_{\mathrm{HU}}$) can achieve the same minimum exergy consumption, and the capital cost may even be lower since the number of stream splittings can be reduced. The determination of expansion inlet temperature ($T_\mathrm{A}$), however, is not graphically straightforward for complex cases where new pinches are created by expansion. This is illustrated in the design procedure presented in Figure 9. The objective of this article is to develop a straightforward design methodology that achieves the target of minimum exergy consumption. The consideration of investment cost as well as retrofit of HENs will be investigated in future work

2. $mc_p(T'_{\mathrm{PI}}-T'_{\mathrm{exp,HU}})\geq Q_{T'_{\mathrm{exp,HU}}}$

A new pinch is created at $T'_{\mathrm{exp,HU}}$ due to a large portion of the stream ($\gamma$) being expanded at $T'_{\mathrm{HU}}$. Expansion at this new pinch (portion $\beta$) is then necessary. The three portions can be determined by the following equations

$$\begin{aligned}
&(mc_p)_\alpha(T'_{\mathrm{PI}}-T'_{\mathrm{exp,HU}})+(mc_p)_\gamma(T'_{\mathrm{PI}}-T'_{\mathrm{exp,HU}})=Q_{T'_{\mathrm{exp,HU}}}\\
&(mc_p)_\alpha(T'_{\mathrm{PI}}-T'_{\mathrm{exp,PI}})+(mc_p)_\beta(T'_{\mathrm{exp,HU}}-T'_{\mathrm{exp,T_{exp,HU}}})\\
&+(mc_p)_\gamma(T'_{\mathrm{PI}}-T'_{\mathrm{exp,HU}})=Q_{\mathrm{CU},0}\\
&(mc_p)_\alpha+(mc_p)_\beta+(mc_p)_\gamma=mc_p
\end{aligned}$$

Similar to the proof for Case (1), it can be proven that the combination of Pinch

Expansion (portion $\alpha$) and expansion at $T'_{\mathrm{HU}}$ (portion $\gamma$) consumes the minimum amount of exergy when the cooling demand is $Q_{\mathrm{CU},0}-Q_\beta$ (accounting for the cooling effect from expansion of portion $\beta$). Since a new pinch is created at $T'_{\mathrm{exp,HU}}$, according to Theorem 1, the cooling demand $Q_\beta$ should be satisfied by expansion at this new pinch (portion $\beta$). The proposed scheme, thus, has the minimum exergy consumption.

If $Q_\gamma$ is large enough, the portion for expansion at $T'_{\mathrm{PI}}$ may vanish and no feasible solutions can be found for the above equations. The three portions are then determined by the following equations

$$\begin{aligned}
&(mc_p)_\alpha=0\\
&(mc_p)_\beta(T'_{\mathrm{exp,HU}}-T'_{\mathrm{exp},T_{\mathrm{exp,HU}}})+Q_{T'_{\mathrm{exp,HU}}}=Q_{\mathrm{CU},0}\\
&(mc_p)_\alpha+(mc_p)_\beta+(mc_p)_\gamma=mc_p
\end{aligned}$$

Since a new pinch is created at $T'_{\mathrm{exp,HU}}$, according to Theorem 2, a combination of expansion at this new pinch and expansion at $T_{\mathrm{HU}}$ has the minimum exergy consumption.

Similar to Case (1), it should be noticed that there are situations where expansion at some intermediate temperature $T_\mathrm{A}$ ($T_{\mathrm{PI}}<T_\mathrm{A}<T_{\mathrm{HU}}$) combined with expansion at $T_{\mathrm{HU}}$ (if necessary) may achieve the same minimum exergy consumption.

**Example 3**. The stream data is shown in Table 7, indicating that a hot stream (H2) is expanded. The outlet temperature of expansion at $T_{\mathrm{HU}}$ is $T_{\mathrm{exp,HU}}=T_{\mathrm{HU}}(p_\mathrm{t}/p_\mathrm{s})^{(n_e-1)/n_e}=151.9°\mathrm{C}>T_0$, thus, Pinch Expansion can reduce the exergy consumption according to Theorem 1. The GCC without pressure manipulation (Case O) is shown in Figure 8a. The pinch temperature is 220/200°C. The outlet temperature of expansion at $T_{\mathrm{PI}}$ is calculated to be $T_{\mathrm{exp,PI}}=38.2°\mathrm{C}$.

The following cases are compared: Case A, expansion at $T_\mathrm{s}$ ($=T_{\mathrm{HU}}$) is used; Case B, Pinch Expansion is used for the entire stream H2; Case C, Pinch Expansion is used for a maximum portion of the stream and the remaining portion is expanded at $T_{\mathrm{HU}}$; Case D, the scheme following Theorem 3 is used; Case E, expansion at an intermediate temperature $T_\mathrm{E}$($T_{\mathrm{PI}}<T_\mathrm{E}<T_{\mathrm{HU}}$) and the cooling demand is satisfied by the expansion. Notice that while cooling demand is completely satisfied by Pinch Expansion in Case C, the same cooling demand in Case D is satisfied by a combination of Pinch Expansion and expansion at $T_{\mathrm{HU}}$. The design of Cases A, B, and C is straightforward. The new stream data for H2 in 5 cases (A–E) is presented in Table 8. The GCCs are presented in Figures 8b–f.

**Table 7. Stream Data for Example 3**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| H1 | 400 | 130 | 2 | 540 | – | – |
| H2 | 400 | 130 | 3 | 810 | 500 | 100 |
| C1 | 200 | 380 | 8 | 1440 | – | – |
| Heat source | 400 | 400 | – | – | – | – |
| Cooling source | 15 | 15 | – | – | – | – |

The original pinch is removed in Case B since the cooling introduced by Pinch Expansion is more than required (450 kW). This also applies to Case C due to the additional cooling below pinch introduced by expansion at $T_{HU}$.

For Case D, since Pinch Expansion produces more cooling than required and $T_{exp,HU} < T_{PI}$, according to Theorem 3, the portion for Pinch Expansion ($\alpha$) should be reduced since cooling is introduced by the expansion of the remaining portion ($\gamma$) at $T_{HU}$. From Figure 8a, $Q_{T'_{exp,HU}}=340.5$ kW. Since $mc_p(T'_{PI}-T'_{exp,HU})=204.3$ kW $< Q_{T'_{exp,HU}}$, no new pinch is created at $T'_{exp,HU}$ according to the discussion about Case (1) for Theorem 3. H2 should, thus, be split into two portions: the portion $\alpha$ is expanded at $T_{PI}$ and the portion $\gamma$ is expanded at $T_{HU}$. The cooling demand is completely satisfied by the expansion of the two portions, that is, $(mc_p)_\alpha(T'_{PI}-T'_{exp,PI})+(mc_p)_\gamma(T'_{PI}-T'_{exp,HU})=Q_{CU,0}$. The two portions can, thus, be determined and are presented in Table 8.

For Case E, assuming that $T_{exp,E}=T_{CU,max}=110$°C , the cooling introduced to the below pinch region is $W_{PI}=mc_p(T_{PI}-T_{exp,E})=3\times(220-110)$ kW$=330$ kW. The value is smaller than the cooling demand (450 kW), and $T_E$ should, thus, be decreased so that the cooling demand can be satisfied. $T_{exp,E}$ is then determined to be $T_{exp,E}=T_{PI}-(Q_{CU,0}/mc_p)=220°C-(450/3)°C=70°C$, and $T_E$ is calculated to be 270.3°C. The cooling demand is satisfied in both Cases D and E, and the original pinch does not change.

The performance comparison is shown in Table 9. The exergy consumption is negative for all the cases, thus, exergy is actually produced from the change of state for the process streams. Compared to Case A, Pinch Expansion of the entire stream (Case B) increases the exergy production by 43.8%. Even more exergy is produced when Pinch Expansion is combined with expansion at $T_{HU}$ (Case C). The exergy production further increases when the scheme following Theorem 3 (Case D) is used. The same values for hot utility demand, expansion work and exergy consumption as in Case D are obtained when expansion at some intermediate temperature $T_E(T_{PI} < T_E < T_{HU})$ is used (Case E). The small differences are caused by round-offs in the calculation. For this simple example, it is easy to determine the expansion scheme for Case E. Since stream splitting is avoided for this particular problem, Case E might be the most attractive alternative when also capital cost is considered.

**Theorem 4.** *For above-ambient processes, if the outlet temperature of expansion at hot utility temperature is lower than ambient temperature, a HEN design with expansion at hot utility temperature consumes the smallest amount of exergy.*

One of the conditions in Theorem 1 for stating that Pinch Expansion consumes the smallest amount of exergy is when

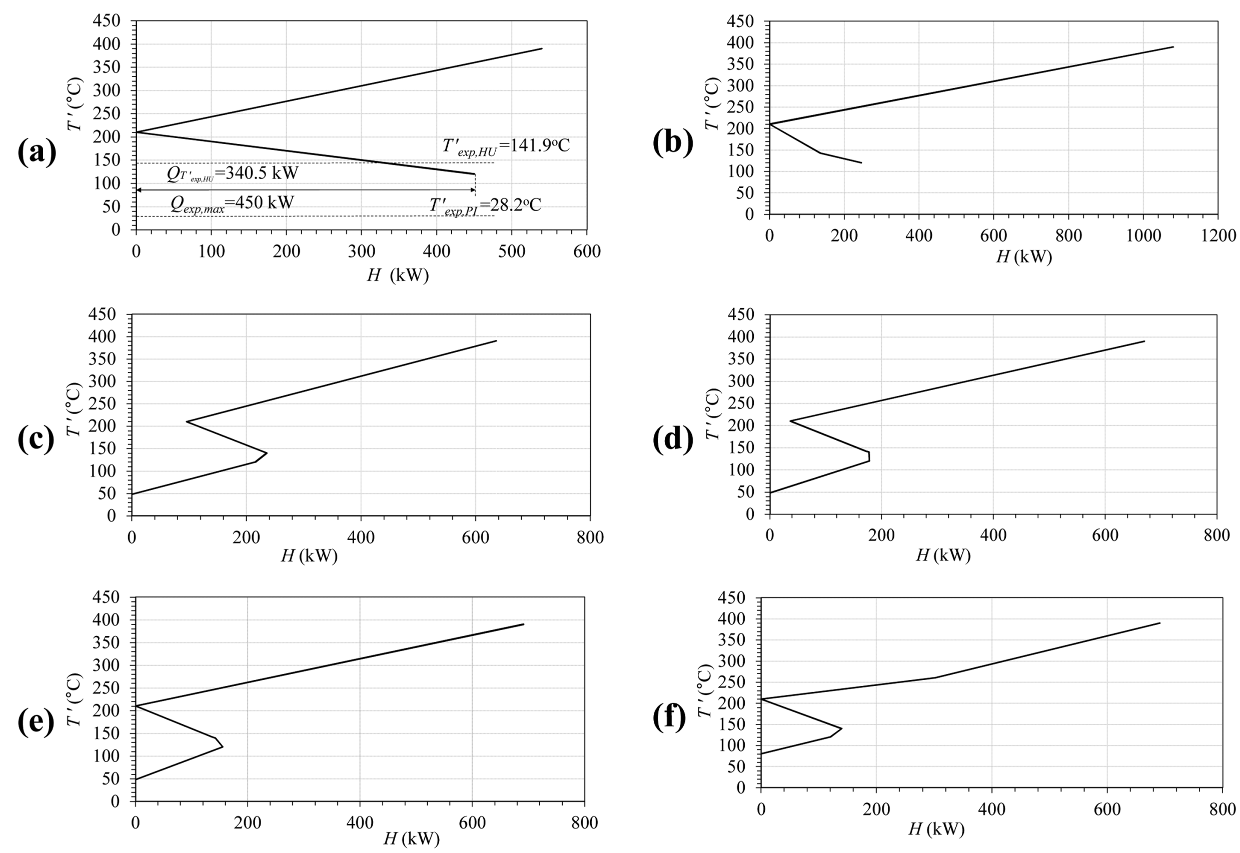


**Figure 8. GCCs for Example 3: (a) Case O without pressure manipulation, (b) Case A, (c) Case B, (d) Case C, (e) Case D, (f) Case E.**

**Table 8. Stream Data for H2 in Example 3**

| Cases | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| **Case A** | | | | | | |
| H2 | 151.9 | 130 | 3 | 65.7 | 100 | 100 |
| **Case B** | | | | | | |
| H2_1 | 400 | 220 | 3 | 540 | 500 | 500 |
| H2_2 | 38.2 | 130 | 3 | 275.4 | 100 | 100 |
| **Case C** | | | | | | |
| H2_α1 | 400 | 220 | 2.48 | 446.4 | 500 | 500 |
| H2_α2 | 38.2 | 130 | 2.48 | 227.7 | 100 | 100 |
| H2_γ | 151.9 | 130 | 0.52 | 11.4 | 100 | 100 |
| **Case D** | | | | | | |
| H2_α1 | 400 | 220 | 2.16 | 388.8 | 500 | 500 |
| H2_α2 | 38.2 | 130 | 2.16 | 198.3 | 100 | 100 |
| H2_γ | 151.9 | 130 | 0.84 | 18.4 | 100 | 100 |
| **Case E** | | | | | | |
| H2_1 | 400 | 270.3 | 3 | 389.1 | 500 | 500 |
| H2_2 | 70 | 130 | 3 | 180 | 100 | 100 |

the expansion at $T_{HU}$ results in an outlet temperature $T_{exp,HU} > T_0$. Theorem 4 deals with the opposite situation when $T_{exp,HU} \leq T_0$. In that case, expansion should not be done at or below the pinch temperature. Two above pinch expansion cases are compared: Case A (expansion at $T_A$) and Case B (expansion at $T_B$). For Case A, $Q_A = Q_{HU,0} + x_A W_A = Q_{HU,0} + mc_p(T_A - T_{PI})$ where $x_A$ is the fraction expanded above pinch, $0 < x_A \leq 1$. Similarly for Case B, $Q_B = Q_{HU,0} + x_B W_B = Q_{HU,0} + mc_p(T_B - T_{PI})$. The difference in exergy consumption can then be expressed as

$$E_A - E_B = (Q_A - Q_B)(1 - T_0/T_{HU}) - (W_A - W_B)$$

$$= mc_p(T_A - T_B)(1 - T_0/T_{HU}) - mc_p(T_A - T_B)[1 - (p_t/p_s)^{(n_e-1)/n_e}]$$

$$= mc_p(T_A - T_B)[(p_t/p_s)^{(n_e-1)/n_e} - T_0/T_{HU}]$$

Since $T_{HU}(p_t/p_s)^{(n_e-1)/n_e} = T_{exp,HU} < T_0$, it can be concluded that $E_A > E_B$ when $T_A < T_B$. This means that a higher expansion temperature reduces exergy consumption, and expansion at $T_{HU}$ consumes the smallest amount of exergy. In practice, however, the pressure ratio can be split (i.e., multistage expansion) so that $T_{exp,HU} > T_0$ and Pinch Expansion can be used.

**Example 4**. The stream data is shown in Table 10. A hot stream (H2) is expanded, and expansion from $T_{HU}$ results in $T_{exp,HU} = T_{HU}(p_t/p_s)^{(n_e-1)/n_e} = -4.8°C < T_0 = 15°C$. Thus, according to Theorem 1, Pinch Expansion should not be used. The pinch temperature for the case without pressure manipulation (Case O) is 220/200°C. The following three cases are compared: Case A, Pinch Expansion is used, $T_A = T_{PI} = 220°C$; Case B, expansion at a temperature between $T_{PI}$ and $T_{HU}$, $T_B = 300°C$; Case C, expansion at $T_C = T_{HU} = 400°C$. The new stream data for stream H2 is

**Table 9. Performance Comparison for Example 3**

| Cases | O | A | B | C | D | E |
|---|---|---|---|---|---|---|
| Hot utility demand (kW) | 540 | 1080 | 635.4 | 669.9 | 691.2 | 690.9 |
| Cold utility demand (kW) | 450 | 245.7 | 0 | 0 | 0 | 0 |
| Pinch temperature (°C) | 210 | 210 | – | – | 210 | 210 |
| Expansion work (kW) | – | 744.3 | 545.4 | 579.0 | 601.1 | 600.9 |
| Exergy consumption (kW) | – | −126.6 | −182.0 | −195.9 | −205.8 | −205.7 |

**Table 10. Stream Data for Example 4**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| H1 | 400 | 60 | 3 | 1020 | – | – |
| H2 | 400 | 280 | 2 | 240 | 2500 | 100 |
| C1 | 200 | 380 | 8 | 1440 | – | – |
| Heat source | 400 | 400 | – | – | – | – |
| Cooling source | 15 | 15 | – | – | – | – |

shown in Table 11. Note that in all 3 cases, hot stream H2 changes identity to a cold stream partly (Cases A and B) or totally (Case C) due to its expansion. The supply temperature for segment H2_2 at −5°C for Cases A and B is after being heated by cold utility from the expander outlet temperature. The performance results are presented in Table 12. As expected, and in line with Theorem 4, expansion at hot utility temperature (Case C) produces the largest amount of exergy.

## Design Procedure

On the basis of the four theorems, a design procedure has been developed and is illustrated in Figure 9 for HEN design including expanders with the objective of minimizing exergy consumption. The first step is to calculate $T_{exp,HU}$ and compare it with $T_0$. According to Theorems 1 and 4, expansion starts at $T_{HU}$ if $T_{exp,HU} < T_0$ and at $T_{PI}$ if $T_{exp,HU} \geq T_0$. When Pinch Expansion is used, the maximum portion that can be expanded, $(mc_p)_{exp,PI,max}$, is limited by the available heat surplus (cooling demand below pinch). Using the concept of Potential Pinch Points, $(mc_p)_{exp,PI,max}$ can be determined. According to Theorem 1, the entire stream is expanded at pinch temperature if its heat capacity flowrate is less than $(mc_p)_{exp,PI,max}$. Otherwise, the stream is split and Pinch Expansion is used for the portion $(mc_p)_{exp,PI,max}$. A new GCC is then produced, where the pressure manipulation of the portion with Pinch Expansion is taken into account. The heating or cooling of the portion from $T_s$ to $T_{PI}$ before expansion and from $T_{exp,PI}$ to $T_t$ after expansion are included. The pressure manipulation of the remaining portion should not be included. The portion available for expansion at the new pinch temperature can then be determined. The procedure is repeated until the entire stream has been expanded or the cooling demand has been completely satisfied (i.e., the pinch problem has become a threshold problem, $(mc_p)_{exp,PI,max} = 0$). In the latter case, according to Theorem 2, the remaining portion of the stream is expanded at either $T_{HU}$ or $T_0$ if $T_{exp,HU} \geq T_{PI}$. Otherwise, and according to Theorem 3, the portion for expansion at the original $T_{PI}$ should be reduced and an iterative procedure is required: A new GCC is produced by including pressure manipulation only for the portion of the stream with expansion

**Table 11. Stream Data for H2 in Example 4**

| Cases | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| **Case A** | | | | | | |
| H2_1 | 400 | 220 | 2 | 360 | 2500 | 2500 |
| H2_2 | −5 | 280 | 2 | 570 | 100 | 100 |
| **Case B** | | | | | | |
| H2_1 | 400 | 300 | 2 | 200 | 2500 | 2500 |
| H2_2 | −5 | 280 | 2 | 570 | 100 | 100 |
| **Case C** | | | | | | |
| H2 | −4.8 | 280 | 2 | 569.6 | 100 | 100 |

**Table 12. Performance Comparison for Example 4**

| Cases | O | A | B | C |
|---|---|---|---|---|
| Hot utility demand (kW) | 660 | 700 | 860 | 1060 |
| Cold utility demand (kW) | 480 | 70 | 70 | 70 |
| Pinch temperatur (°C) | 210 | 210 | 210 | 210 |
| Expansion work (kW) | – | 593.2 | 689.4 | 809.6 |
| Exergy consumption (kW) | – | −192.8 | −197.5 | −203.3 |

at $T_{HU}$, and the procedure for implementing Pinch Expansion is used. The procedure stops if the entire stream has been expanded, otherwise the portion for expansion at $T_{HU}$ increases until $T_{exp,HU}$ is higher than the new pinch temperature (which means that the pinches above $T_{exp,HU}$ have been removed). When the cooling demand has been completely satisfied by expansion at the new pinch point(s) below $T_{exp,HU}$, the remaining portion of the stream ($\gamma$) should be expanded at $T_{HU}$ or $T_0$.

According to the design procedure, it can be observed that to achieve a design with minimum exergy consumption, expansion of streams should be implemented at pinch temperatures, hot utility temperature, or ambient temperature. In some cases, the same minimum exergy consumption may also be achieved when expansion at some intermediate temperature ($T_{PI} < T < T_{HU}$) is included. However, the determination of the expansion inlet temperature is not straightforward. The design procedure is further illustrated by Example 5 and 6. The cooling demand is not satisfied by using Pinch Expansion in Example 5, while it is completely satisfied in Example 6 and the cooling resulting from expansion at hot utility temperature is used to reduce the portions using Pinch Expansion.

**Example 5**. The stream data is shown in Table 13. The outlet temperature of expansion (H1) at $T_{HU}$ is calculated to be $T_{exp,HU}=T_{HU}(p_t/p_s)^{(n_e-1)/n_e}=218.7°C > T_0=15°C$. Thus, Pinch Expansion should be used. The GCC without pressure manipulation (Case O) is shown in Figure 10a. The pinch temperature is 330/310°C. The outlet temperature of expansion at $T_{PI}$ is calculated to be $T_{exp,PI}=167.5°C$. The following three cases are compared: Case A, expansion at $T_A=T_{HU}=T_s=400°C$; Case B, expansion at $T_B=T_{PI}=330°C$; Case C, the proposed procedure (see Figure 9) is applied. The design of Cases A and B is straightforward. The new stream data for H1 is presented in Table 14. The GCCs are shown in Figures 10b, c. In Cases A and B, the pinch changes due to enthalpy change of the stream subject to expansion. Pinch Expansion for the entire stream (Case B) increases the hot utility demand (compared to Case O).

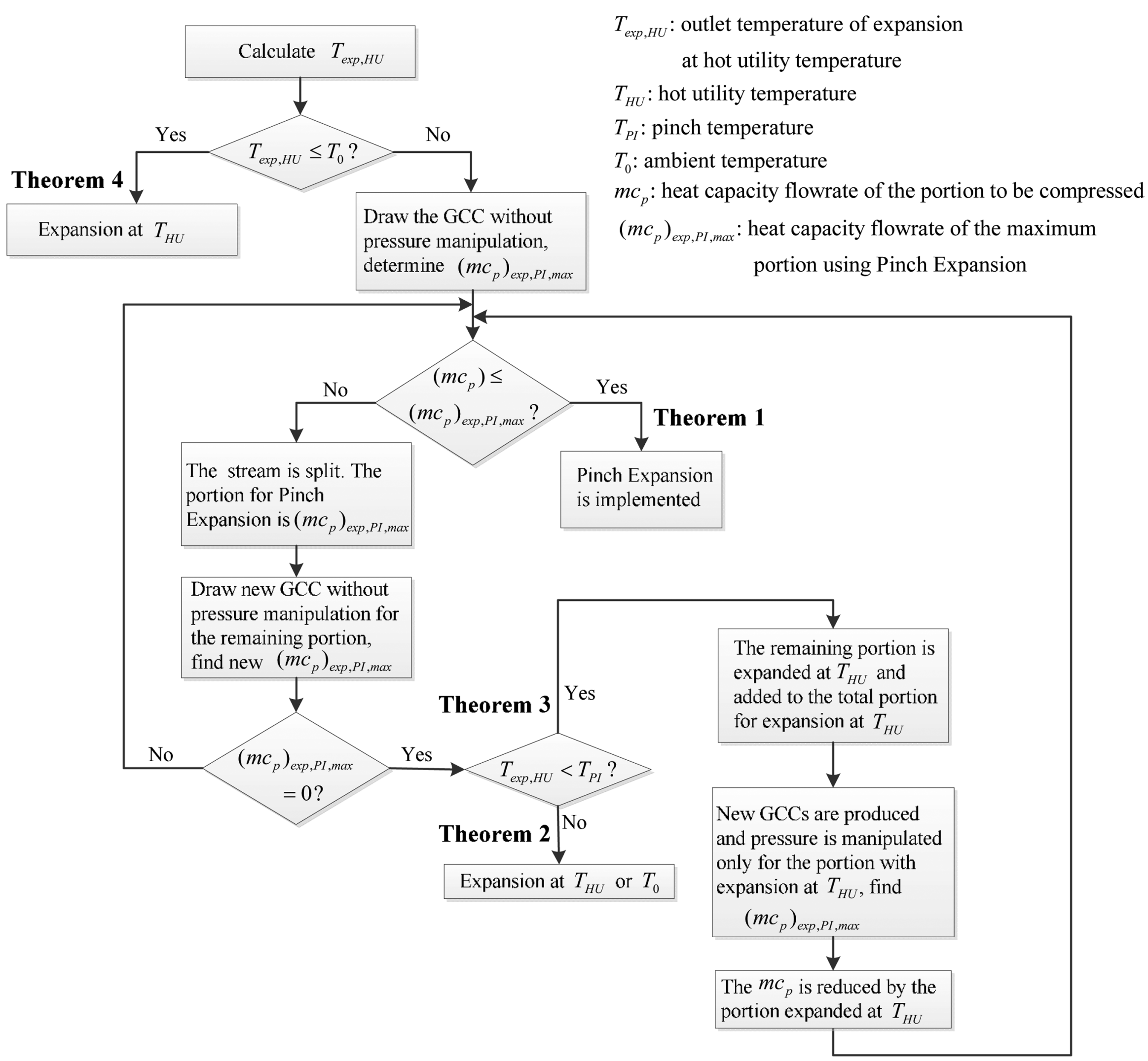


**Figure 9. Design procedure for integrating expanders into HENs.**

**Table 13. Stream Data for Example 5**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| H1 | 400 | 60 | 3 | 1020 | 300 | 100 |
| H2 | 330 | 80 | 9 | 2250 | – | – |
| C1 | 15 | 220 | 6 | 1230 | – | – |
| C2 | 140 | 380 | 8 | 1920 | – | – |
| Heat source | 400 | 400 | – | – | – | – |
| Cooling source | 15 | 15 | – | – | – | – |

This is in line with Theorem 1 indicating that Pinch Expansion can not be fully utilized when such expansion removes the original pinch point.

For Case C, based on the GCC shown in Figure 10a, the maximum work resulting from Pinch Expansion is determined to be $Q_{\mathrm{exp,max}}=$ 200 kW by using the concept of Potential Pinch Points. The maximum portion of the stream that can be expanded is then determined to be: $(mc_p)_{\mathrm{exp,PI,max}}=Q_{\mathrm{exp,max}}/(T'_{\mathrm{PI}}-T'_{\mathrm{exp,PI}})=200\ \mathrm{kW}/(320°\mathrm{C}-157.5°\mathrm{C})=1.23\ \mathrm{kW}/°\mathrm{C}$. Thus, stream H1 is split into two portions: the first portion ($\alpha$) uses Pinch Expansion and the expansion of the second portion ($\beta$) is investigated later. The corresponding GCC (Case C1) is shown in Figure 10d. The GCC now has two pinch points. The new pinch temperature ($T'_{\mathrm{PI,new}}$) is 150°C. The outlet temperature of expansion at $T_{\mathrm{PI,new}}$ is calculated to be $T_{\mathrm{exp,PI,new}}=$ 43.3°C. The maximum portion of part $\beta$ of stream H1 that can be expanded at $T_{\mathrm{PI,new}}$ is determined to be: $(mc_p)_{\mathrm{exp,PI,max,new}}=Q_{\mathrm{exp,max,new}}/(T'_{\mathrm{PI,new}}-T'_{\mathrm{exp,PI,new}})=270\ \mathrm{kW}/(150°\mathrm{C}-33.3°\mathrm{C})=2.31\ \mathrm{kW}/°\mathrm{C}$. Thus, all the remaining portion ($\beta=1.77$ kW/°C) can be expanded at the new pinch. The corresponding GCC (Case C2) is shown in Figure 10e. The hot utility demand does not change for Cases C1 and C2 compared to Case O.

**Table 14. Stream Data for H1 in Example 5**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| **Case A** | | | | | | |
| H1 | 218.7 | 60 | 3 | 476.1 | 100 | 100 |
| **Case B** | | | | | | |
| H1_1 | 400 | 330 | 3 | 210 | 300 | 300 |
| H1_2 | 167.5 | 60 | 3 | 322.5 | 100 | 100 |
| **Case C1** | | | | | | |
| H1_$\alpha$1 | 400 | 330 | 1.23 | 86.1 | 300 | 300 |
| H1_$\alpha$2 | 167.5 | 60 | 1.23 | 132.2 | 100 | 100 |
| H1_$\beta$ | 400 | 60 | 1.77 | 601.8 | 300 | 100 |
| **Case C2** | | | | | | |
| H1_$\alpha$1 | 400 | 330 | 1.23 | 86.1 | 300 | 300 |
| H1_$\alpha$2 | 167.5 | 60 | 1.23 | 132.2 | 100 | 100 |
| H1_$\beta$1 | 400 | 160 | 1.77 | 424.8 | 300 | 300 |
| H1_$\beta$2 | 43.3 | 60 | 1.77 | 29.6 | 100 | 100 |

Detailed heat exchanger networks for Cases A, B and C are illustrated in Figures 11a–c. Since the objective of this article is to illustrate the design of HENs with minimum exergy consumption, the design evolution stage using loops and paths to remove units with small duties is not included. Also, stream splits are not optimized, only adjusted to achieve sufficient driving forces $\Delta T \geq \Delta T_{\mathrm{min}}$. The performance data is shown in Table 15. Exergy is actually produced from the process streams (i.e., negative exergy consumption). Compared to expansion at $T_{\mathrm{HU}}$ (Case A), direct Pinch Expansion (Case B) increases exergy consumption (or reduces exergy production) by 16.4% and uses 1 more heat exchanger. As a result of Pinch Expansion combined with stream splitting, the exergy

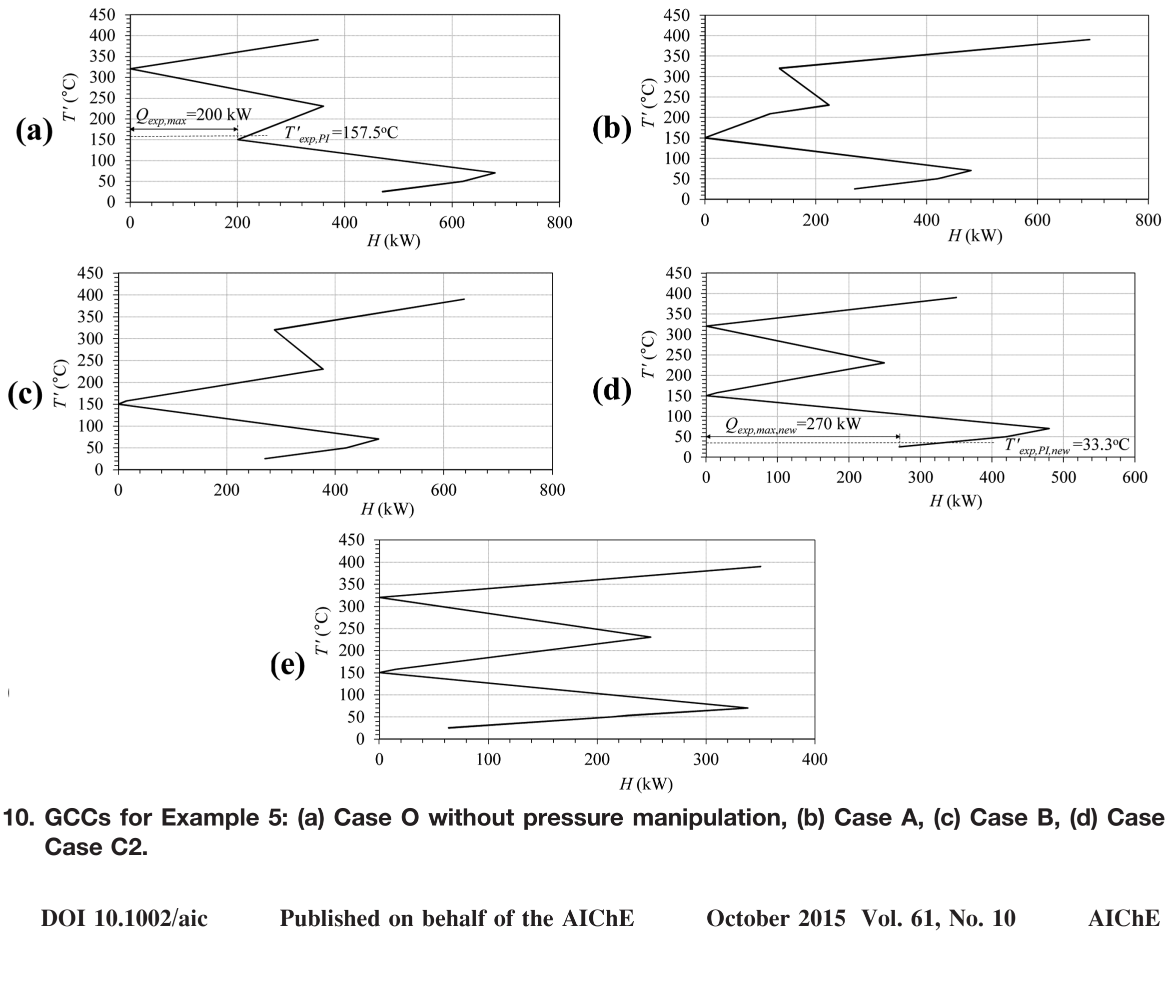


**Figure 10. GCCs for Example 5: (a) Case O without pressure manipulation, (b) Case A, (c) Case B, (d) Case C1, (e) Case C2.**

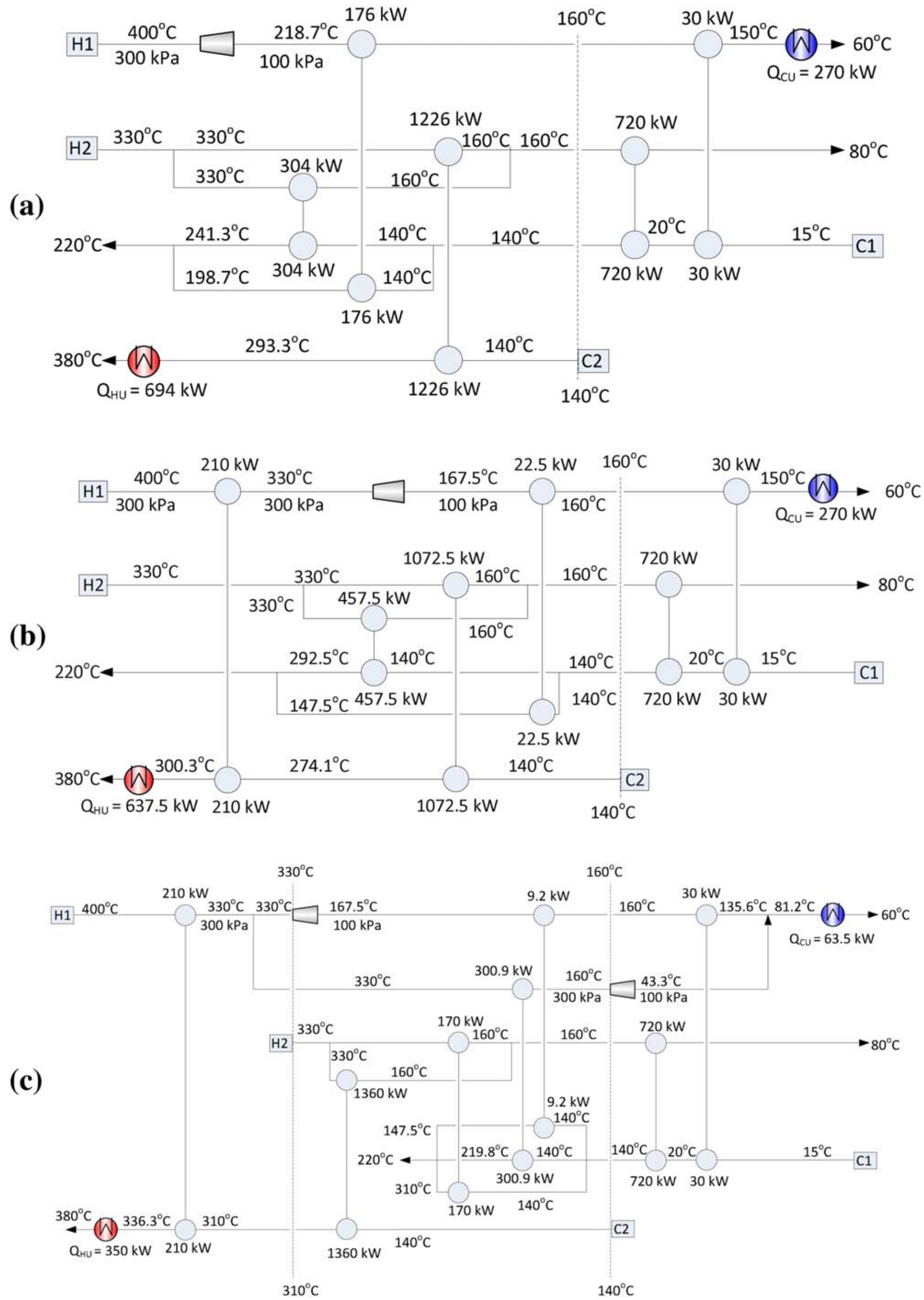


**Figure 11. Heat exchanger networks for Example 5: (a) Case A, (b) Case B, (c) Case C.**

**[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]**

production of Case C is increased by 40.4% compared to Case A, however, 2 more heat exchangers and 1 more expander are used. When comparing Case C (optimal placement of expanders) with Case O (design without pressure manipulation), it can be observed that hot utility demand is the same, and the sum of cold utility demand and expansion work in Case C is equal to the cold utility demand in Case O. This shows that the reduced heat surplus below pinch (i.e., cooling demand) is completely converted into power, and the expander is appropriately placed. In this case, one stream split was required to fully utilize Pinch Expansion. In other cases, Pinch Expansion can only be partly utilized, and one has to resort to Ambient Expansion or expansion at hot utility temperature.

EXAMPLE 6. The stream data is shown in Table 16. The GCC without pressure manipulation (Case O) is the same as the one shown in Figure 10a. The change from Example 5 is

**Table 15. Performance Comparison for Example 5**

| Cases | O | A | B | C (= C2) |
|---|---|---|---|---|
| Hot utility demand (kW) | 350 | 694 | 637.5 | 350 |
| Cold utility demand (kW) | 470 | 270 | 270 | 63.5 |
| Pinch temperature (°C) | 320 | 150 | 150 | 320;150 |
| Expansion work (kW) | – | 543.9 | 487.5 | 406.6 |
| Exergy consumption (kW) | – | −147.0 | −122.9 | −206.4 |
| Number of heat exchangers | – | 7 | 8 | 9 |
| Number of expanders | – | 1 | 1 | 2 |

**Table 16. Stream Data for Example 6**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| H1 | 400 | 60 | 6 | 2040 | 300 | 100 |
| H2 | 330 | 80 | 9 | 2250 | – | – |
| C1 | 15 | 220 | 6 | 1230 | – | – |
| C2 | 140 | 380 | 8 | 1920 | – | – |
| C3 | 40 | 380 | 3 | 1020 | – | – |
| Heat source | 400 | 400 | – | – | – | – |
| Cooling source | 15 | 15 | – | – | – | – |

**Table 17. Stream Data for H1 in Example 6**

| Stream | $T_s$ (°C) | $T_t$ (°C) | $mc_p$ (kW/°C) | $\Delta H$ (kW) | $p_s$ (kPa) | $p_t$ (kPa) |
|---|---|---|---|---|---|---|
| **Case A** | | | | | | |
| H1_$\alpha$1 | 400 | 330 | 1.23 | 86.1 | 300 | 300 |
| H1_$\alpha$2 | 167.5 | 60 | 1.23 | 132.2 | 100 | 100 |
| H1_$\beta$1 | 400 | 160 | 2.31 | 424.8 | 300 | 300 |
| H1_$\beta$2 | 43.3 | 60 | 2.31 | 29.6 | 100 | 100 |
| H1_$\gamma$ | 218.7 | 60 | 2.46 | 390.4 | 100 | 100 |
| **Case B1** | | | | | | |
| H1_O | 400 | 60 | 3.54 | 1203.6 | 300 | 300 |
| H1_$\gamma$ | 218.7 | 60 | 2.46 | 390.4 | 100 | 100 |
| **Case B2** | | | | | | |
| H1_$\beta$1 | 400 | 160 | 2.31 | 424.8 | 300 | 300 |
| H1_$\beta$2 | 43.3 | 60 | 2.31 | 29.6 | 100 | 100 |
| H1_$\gamma$ | 218.7 | 60 | 3.69 | 585.6 | 100 | 100 |

that H1 has doubled its $mc_p$ value and a new cold stream (C3) is introduced that exactly matches the change of H1 from a GCC point of view. According to the study in Example 5, to fulfill the cooling demand, a first portion ($\alpha = 1.23$ kW/°C) of stream H1 is expanded at the original pinch temperature (330°C), and a second portion ($\beta = (mc_p)_{\text{exp,PI,max,new}} =$ 2.31 kW/°C) is expanded at the new pinch temperature (160°C). After the cooling demand has been satisfied, there is still a remaining portion of H1 ($\gamma$=2.46 kW/°C) to be expanded. Example 5 shows that $T_{\text{exp,HU}}=218.7$°C $< T_{\text{PI}}=330$°C. The following two cases are investigated: Case A, the portion $\gamma$ is expanded at $T_{\text{HU}}$ and the expansions for portions $\alpha$ and $\beta$ do not change from Example 5; Case B, the expansions for portions $\alpha$, $\beta$, and $\gamma$ change according to Theorem 3 and the procedure presented in Figure 9. For Case A, the procedure is straightforward. The new stream data for H1 is shown in Table 17 and the GCC is shown in Figure 12a.

For Case B, the iterative procedure presented in Figure 9 is required. The first subcase (B1) is to draw the new GCC including pressure manipulation only for the remaining portion $\gamma$=2.46 kW/°C. The new pinch temperature ($T'_{\text{PI,new}}$) is 150°C. The maximum portion of part $\beta$ of stream H1 that can be expanded at $T'_{\text{PI,new}}$ is determined to be $(mc_p)_{\text{exp,PI,max,new}}=$ 2.31 kW/°C (see calculations in Example 5). The portion without pressure manipulation (referred to as H1_O in Table 17) is 3.54 kW/°C. Thus, there is still a remaining portion ($3.54 - 2.31 = 1.23$ kW/°C) to be expanded after the portion $\beta$=2.31 kW/°C has been expanded at $T'_{\text{PI,new}}$. This remaining portion is added to the portion $\gamma$ and, thus, $\gamma$=3.69 kW/°C. The second subcase (B2) is then straightforward. The stream data for H1 in Cases B1 and B2 are shown in Table 17 and the GCCs are shown Figures 12b,c. The original pinch point (at $T'$=320°C, see Figure 10a for Example 5) disappears due to a large portion $\gamma$ that is expanded at $T_{\text{HU}}$.

The performance comparison is shown in Table 18. The cooling demand in both Cases A and B has been completely satisfied. The exergy production is larger for Case B than Case A. The reason is that the pinch has been changed by expanding the portion at $T_{\text{HU}}$. Any expansion at the original pinch temperature is actually performed above the new pinch temperature. Such expansion should be avoided according to Theorem 1. Note that Case B consumes 23.1 kW more heat, however, the same amount of additional work (23.4 kW) is produced. The small difference (0.3 kW) is due to round-off errors when calculating the $mc_p$ of the stream splits. This is in line with the previous statement that expansion at $T_{\text{HU}}$ is preferred since the heat is completely converted into work (see the interpretations of Theorem 2).

## Discussions

Following the graphical design procedure presented in Figure 9, minimum exergy consumption or maximum exergy production can be achieved at an early stage of process

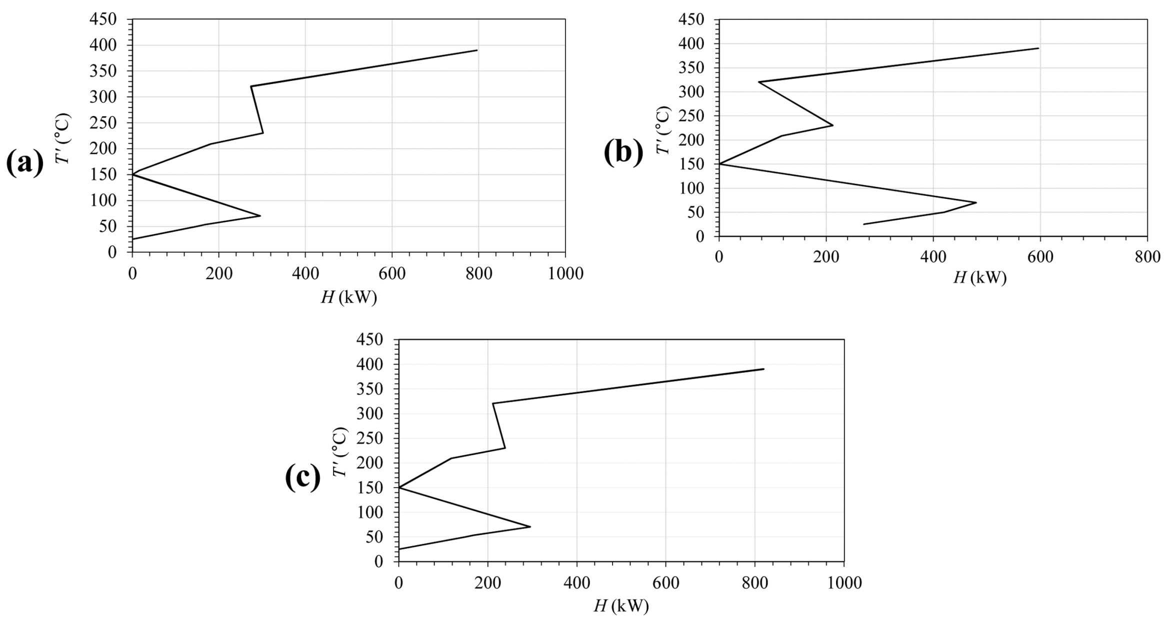


**Figure 12. GCCs for Example 6: (a) Case A, (b) Case B1, (c) Case B2.**

Table 18. Performance Comparison for Example 6

| Cases | O | A | B (= B2) |
|---|---|---|---|
| Hot utility demand (kW) | 350 | 795.9 | 819.0 |
| Cold utility demand (kW) | 470 | 0 | 0 |
| Pinch temperature (°C) | 320 | 150 | 150 |
| Expansion work (kW) | – | 915.2 | 938.6 |
| Exergy consumption (kW) | – | −460 | −470.2 |

design under the assumptions defined in the Section "Problem statement". The expander efficiency has been assumed to be $\eta_{\infty,\exp}=1$ in the examples, however, this assumption was not used in the theorems (see the proofs). The design procedure can, thus, be applied for cases with lower expander efficiencies as long as the expander efficiency is a constant. Since the definition of exergy assumes reversible processes for the conversion of heat into work, exergy may, thus, not be a good parameter to balance the complex trade-off between heat and work in real processes. Alternatively, an equivalent hot utility temperature ($T_{\mathrm{eq,HU}}$) can be used instead of the real $T_{\mathrm{HU}}$ in the theorems and design procedure. Once a conversion factor ($\lambda$) between work and heat is defined (normally known as thermal efficiency), the equivalent temperature can be determined from the following equation: $\lambda=1-T_0/T_{\mathrm{eq,HU}}$.

Although only one stream is assumed to be expanded in one stage in this article, the procedure can be extended to include multiple streams and expansion stages. The temperature driving forces for heat transfer between expanded streams and other process streams can be reduced by manipulating the pressure ratio. When the pressure ratios are similar in value, the influence on exergy consumption of the integration sequence for different expansion stages with the HEN is negligible. If the pressure ratios are quite different, energy savings can be achieved by splitting the pressure ratios. The optimal distribution of pressure ratios among multiple expansion stages, the integration of both expanders and compressors into HENs for above and below ambient processes, as well as industrial applications will be presented in future publications.

This article focuses on fundamental insights about above ambient HEN design including expanders. The following work is to be performed in the future for a more comprehensive study of the topic:

1. Multiple utilities are included. It has been assumed that one hot utility with constant temperature is used for heat recovery problems in this article. In many applications, variable temperature utilities as well as multiple utilities are used and should, thus, be studied.
2. Cost should be considered. The possibilities of removing small units at the expense of more energy consumption should be investigated.
3. Mathematical optimization models can be built. The design methodology presented in this article can be easily applied to small size problems. For large scale industrial applications, mathematical optimization models may be more efficient to find the optimum (the most energy efficient or cost-effective alternatives). This study provides useful insights for the building and solution of such optimization models.

## Conclusions

A systematic procedure for above ambient heat exchanger network design including expanders has been developed. Since both heat and work are involved, the objective has been chosen to be minimum exergy consumption. Four theorems are proposed and proven under certain well-defined conditions to assist the design procedure and they can be simplified into the following three statements:

1. Pinch Expansion is preferred if the outlet temperature of expansion at hot utility temperature is higher than ambient temperature (Theorem 1), otherwise expansion should start at hot utility temperature (Theorem 4);
2. Expansion at hot utility temperature or ambient temperature is used after the cooling demand has been completely satisfied by Pinch Expansion provided that the outlet temperature of expansion at hot utility temperature is higher than the pinch temperature (Theorem 2); otherwise
3. The cooling resulting from expansion at hot utility temperature should be utilized to reduce the portion with Pinch Expansion (Theorem 3).

In cases where the original pinch point is removed by using Pinch Expansion, maximum use of Pinch Expansion is obtained by stream splitting. It is concluded that to achieve a HEN design with minimum exergy consumption (or maximum exergy production), expansion should be done at pinch temperatures, hot utility temperature, or ambient temperature. No other expansion temperature will improve exergy consumption/production.

A straightforward graphical design procedure based on the Grand Composite Curve is presented. A small error, $ymc_p\Delta T_{\min}$ where $0 \le y \le 1$, is introduced in the procedure by not considering stream type (hot/cold) and the location of temperatures (supply/target). The results may, thus, move slightly away from the optimum (i.e., minimum exergy consumption), however, the advantage is that traditional Grand Composite Curves can be used directly. The complexity of the design procedure is, thus, considerably reduced.


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## Notation

### *Roman letters*

$A$ = heat transfer area, m$^2$
$c_p$ = specific heat capacity at constant pressure, kJ/(kg°C)
$c_v$ = specific heat capacity at constant volume, kJ/(kg°C)
$C$ = cost
$E$ = exergy, kW
$h$ = heat-transfer coefficient for process streams, kW/(°Cm$^2$)
$H$ = enthalpy, kW
$m$ = mass flow rate, kg/s
$n_e$ = polytropic index for expansion
$p$ = pressure, kPa
$Q$ = heat, kW
$T$ = temperature, °C or K
$T'$ = modified temperature, °C or K
$\Delta T_{\min}$ = minimum temperature difference, °C or K
$U$ = overall heat-transfer coefficient, kW/(°Cm$^2$)
$W$ = work, kW
$x$ = fraction
$y$ = fraction

### Greek letters

$\alpha$ = portion of stream being expanded at the original pinch temperature
$\beta$ = portion of stream being expanded at the new pinch temperature
$\gamma$ = portion of stream being expanded at ambient temperature
$\eta_{\infty,\exp}$ = expander polytropic efficiency
$\kappa$ = specific heat ratio
$\lambda$ = conversion factor between work and heat

### Subscripts

exp = expansion
EXP = expander
eq = equivalent
HE = heat exchanger
HU = hot utility
CU = cold utility
LM = logarithmic mean
max = maximum
min = minimum
PI = pinch
s = supply
t = target
0 = ambient; original

### Abbreviations

CAPEC = capital cost
CC = composite curve
CEPCI = chemical engineering plant cost index
ExPAnD = extended pinch analysis and design procedure
GCC = grand composite curve
HEN = heat exchanger network
MINLP = mixed integer nonlinear programming
OPEX = operating cost
TAC = total annualized cost

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## Appendix A: Influences of $\Delta T_{\min}$ on Hot Utility Demand

The Grand Composite Curve (GCC) has been used as an important tool in a systematic procedure for determining the appropriate placement of expanders when integrated with heat exchanger networks above ambient temperature. The purpose of this appendix is to illustrate and quantify the effect of not paying attention to the type of streams (hot/cold) to be expanded and the relative position of stream temperatures when using the GCC. The appendix focuses on cases where Pinch Expansion can be fully or partly utilized to minimize exergy consumption (or maximize exergy production).

The temperatures of interest are then the supply ($T_s$) and target ($T_t$) temperatures of the stream to be expanded, the pinch temperature ($T_{PI}$), and the outlet temperature from an expander ($T_{exp,PI}$) operating with $T_{PI}$ as the inlet temperature. With 4 different temperatures for a stream there are (4!)=24 distinctly different sequences of these temperatures. This set can, however, be reduced since the relative position of $T_s$ and $T_t$ is given by the stream type, and $T_{exp,PI}$ is always less than $T_{PI}$ (expansion reduces the temperature of a stream). The total number of cases that needs to be investigated is then $24/(2\times2)=6$ for cold streams and the same number for hot streams.

For cold streams the cases to be analyzed are

C.1: $T_t > T_s \geq T_{PI} > T_{exp,PI}$
C.2.a: $T_s < T_{PI}$, $T_t \geq T_{PI}$, $T_s \geq T_{exp,PI}$
C.2.b: $T_s < T_{PI}$, $T_t \geq T_{PI}$, $T_s < T_{exp,PI}$
C.3.a: $T_s < T_{PI}$, $T_t < T_{PI}$, $T_t > T_{exp,PI}$, $T_s \geq T_{exp,PI}$
C.3.b: $T_s < T_{PI}$, $T_t < T_{PI}$, $T_t > T_{exp,PI}$, $T_s < T_{exp,PI}$
C.4: $T_s < T_t \leq T_{exp,PI} < T_{PI}$

**Table A1. Selection Criteria for Case Studies**

| Cases | Recuperative heating | Heat deficit |
|---|---|---|
| C.1 | Yes | Yes (the heat deficit is caused by recuperative heating) |
| C.3.a | No | No |
| H.1 | Yes | Yes (the heat deficit is caused by recuperative heating) |
| H.2.b | No | No |
| H.4.b | Yes (for Case H.4.b.i); No (for Case H.4.b.ii) | Yes (the heat deficit is caused by the need to heat a hot stream to $T_{PI}$) |

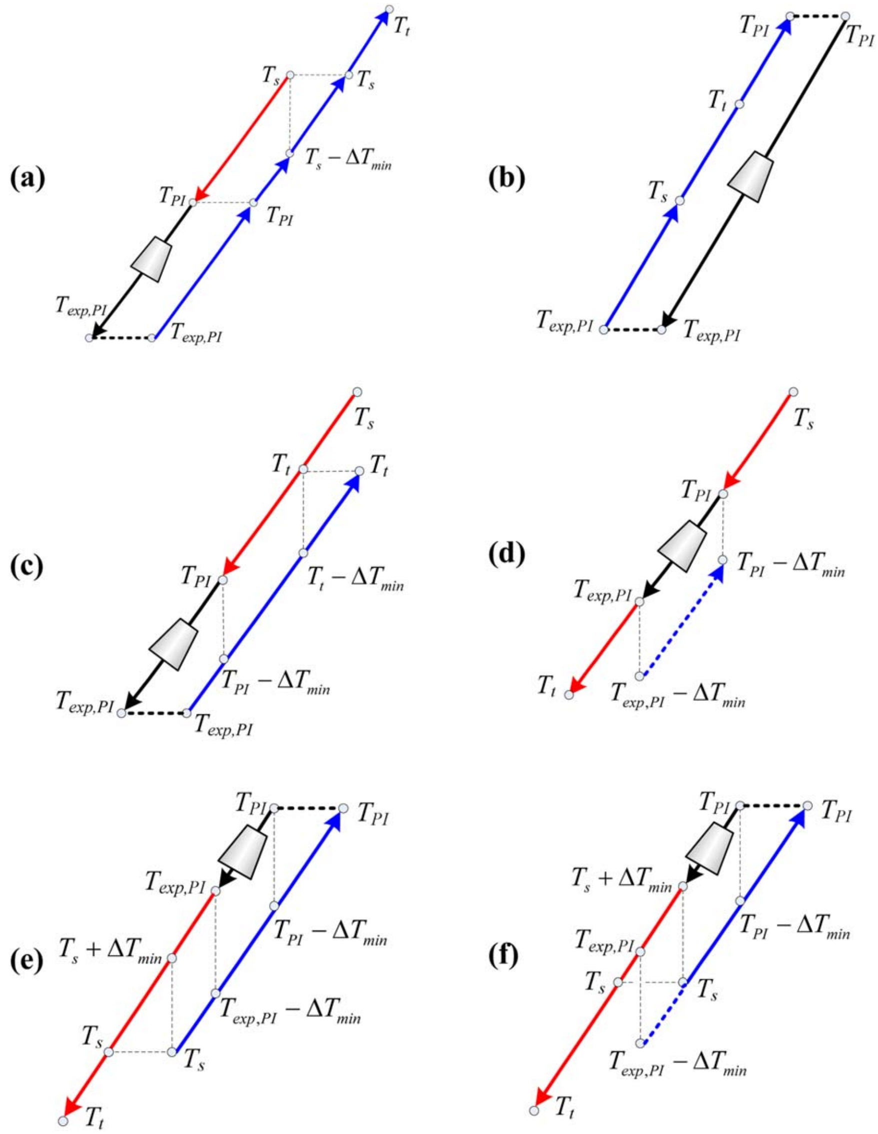


**Figure A1. Illustration of heat balance: (a) Case C.1, (b) Case C.3.a, (c) Case H.1, (d) Case H.2.b, (e) Case H.4.b.i, (f) Case H.4.b.ii.**

**[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]**

Similarly for hot streams, the following cases should be analyzed

H.1: $T_{\mathrm{s}} \geq T_{\mathrm{PI}}$, $T_{\mathrm{t}} \geq T_{\mathrm{PI}}$

H.2.a: $T_{\mathrm{s}} \geq T_{\mathrm{PI}}$, $T_{\mathrm{t}} < T_{\mathrm{PI}}$, $T_{\mathrm{t}} > T_{\mathrm{exp,PI}}$

H.2.b: $T_{\mathrm{s}} \geq T_{\mathrm{PI}}$, $T_{\mathrm{t}} < T_{\mathrm{PI}}$, $T_{\mathrm{t}} \leq T_{\mathrm{exp,PI}}$

H.3: $T_{\mathrm{s}} < T_{\mathrm{PI}}$, $T_{\mathrm{t}} > T_{\mathrm{exp,PI}}$

H.4.a: $T_{\mathrm{s}} < T_{\mathrm{PI}}$, $T_{\mathrm{t}} \leq T_{\mathrm{exp,PI}}$, $T_{\mathrm{s}} > T_{\mathrm{exp,PI}}$

H.4.b: $T_{\mathrm{s}} < T_{\mathrm{PI}}$, $T_{\mathrm{t}} < T_{\mathrm{exp,PI}}$, $T_{\mathrm{s}} \leq T_{\mathrm{exp,PI}}$

Among the 12 cases, the following five typical cases are selected and studied in detail to illustrate all relevant situations that can be encountered: C.1, C.3.a, H.1, H.2.b, and H.4.b. The selection criteria (1) whether recuperative heating is used, and (2) whether a heat deficit is caused by using Pinch Expansion, are shown in Table A1. For cold streams, there is a heat deficit whenever recuperative heating is used. However, for hot streams, a heat deficit may be caused by either recuperative heating or by the need to heat the stream to $T_{\mathrm{PI}}$. For all the five cases discussed in the following, the portion of the stream to be expanded at $T_{\mathrm{PI}}$ is limited to $(mc_p)_{\mathrm{exp,PI,max}}$ as illustrated by Figure 2 and the discussion following Theorem 1. The reason is that the heat surplus below pinch is limited.

For Case C.1, the stream is expanded after being cooled from $T_{\mathrm{s}}$ to $T_{\mathrm{PI}}$, and then heated from $T_{\mathrm{exp,PI}}$ to $T_{\mathrm{t}}$. Figure A1a illustrates the operation of the stream. The heating of the stream from $T_{\mathrm{exp,PI}}$ to $T_{\mathrm{PI}}$ is satisfied by the heat surplus below pinch that represents a limit for the portion of the stream that can be subject to Pinch Expansion, $(mc_p)_{\mathrm{exp,PI,max}}$. A part of the heating from $T_{\mathrm{PI}}$ to $T_{\mathrm{s}}$ can be satisfied by the cooling of the stream from $T_{\mathrm{s}}$ to $T_{\mathrm{PI}}$ before expansion (recuperative heating). Due to the need for driving forces, however, the cold stream can only be heated to $T_{\mathrm{s}} - \Delta T_{\mathrm{min}}$. Thus, there is a heat deficit of $mc_p \Delta T_{\mathrm{min}}$ when $T_{\mathrm{s}} - \Delta T_{\mathrm{min}} \geq T_{\mathrm{PI}}$ and $ymc_p \Delta T_{\mathrm{min}}$ where $0 \leq y < 1$ when $T_{\mathrm{s}} - \Delta T_{\mathrm{min}} < T_{\mathrm{PI}}$. Notice that in the latter case, recuperative heating can no longer be utilized, but the heat deficit is gradually reduced to zero as $T_{\mathrm{s}}$ approaches $T_{\mathrm{PI}}$.

For Case C.3.a, the stream is expanded after being heated from $T_{\mathrm{s}}$ to $T_{\mathrm{PI}}$, and then heated from $T_{\mathrm{exp,PI}}$ to $T_{\mathrm{t}}$, as shown in Figure A1b. The stream is, thus, actually heated twice between $T_{\mathrm{s}}$ and $T_{\mathrm{t}}$. The heating from $T_{\mathrm{exp,PI}}$ to $T_{\mathrm{PI}}$ is covered by the heat surplus below pinch, and the additional heating from $T_{\mathrm{s}}$ to $T_{\mathrm{t}}$ is part of the original GCC used to determine $Q_{\mathrm{HU,0}}$. There is, thus, no heat deficit caused by using Pinch Expansion in this case.

For Case H.1, the stream is expanded after being cooled from $T_s$ to $T_{PI}$, and then heated from $T_{exp,PI}$ to $T_t$, as shown in Figure A1c. Note that the heat surplus below pinch can be used to heat the stream up to the pinch temperature for cold streams, that is, $T_{PI}-\Delta T_{min}$ where $T_{PI}$ is the pinch temperature for hot streams. When $T_{exp,PI} \leq T_{PI}-\Delta T_{min}$, the stream can, thus, be heated from $T_{exp,PI}$ to $T_{PI}-\Delta T_{min}$ after expansion. The heating of the stream from $T_{PI}-\Delta T_{min}$ to $T_t-\Delta T_{min}$ can be satisfied by the cooling of the same stream from $T_t$ to $T_{PI}$ before expansion (recuperative heating). The further heating of the stream from $T_t-\Delta T_{min}$ to $T_t$ introduces a heat deficit of $mc_p\Delta T_{min}$. When $T_{exp,PI} > T_{PI}-\Delta T_{min}$, the stream is heated from $T_{exp,PI}$ to $T_t-\Delta T_{min}$ by recuperative heating. The heat available from cooling the stream from $T_{exp,PI}+\Delta T_{min}$ to $T_{PI}$ before expansion now represents additional heat that can be used for heating other cold streams and, thus, reduce the hot utility demand. The heat deficit is, thus, $ymc_p\Delta T_{min}$ where $0 < y < 1$.

For Case H.2.b, the stream is expanded after being cooled from $T_s$ to $T_{PI}$, and then further cooled from $T_{exp,PI}$ to $T_t$, as shown in Figure A1d. The heat available from cooling the stream from $T_s$ to $T_t$ is included when using the GCC (without pressure manipulation) to determine $Q_{HU,0}$. This heat is actually still available when Pinch Expansion is used. The cooling of the stream from $T_s$ to $T_{PI}$ (before expansion) and from $T_{exp,PI}$ to $T_t$ (after expansion) does not change. The heat resulting from cooling the stream from $T_{PI}$ to $T_{exp,PI}$ is not available since the stream is expanded instead of being cooled, however, since the portion for Pinch Expansion is limited, this heat can be covered by the heat surplus below pinch. As a result, there is no heat deficit caused by using Pinch Expansion in this case.

For Case H.4.b, the stream is expanded after being heated from $T_s$ to $T_{PI}$, and then cooled from $T_{exp,PI}$ to $T_t$. This case is analyzed in two subcases: $T_s \leq T_{exp,PI}-\Delta T_{min}$ (Case H.4.b.i) and $T_s > T_{exp,PI}-\Delta T_{min}$ (Case H.4.b.ii), as shown in Figures A1e, f. For Case H.4.b.i, the heating of the stream from $T_s$ to $T_{exp,PI}-\Delta T_{min}$ is satisfied by recuperating heating, that is, the cooling of the stream from $T_{exp,PI}$ to $T_s+\Delta T_{min}$ after expansion. The heating of the stream from $T_{exp,PI}-\Delta T_{min}$ to $T_{PI}-\Delta T_{min}$ can be covered by the heat surplus below pinch. Note that, as stated in the analysis of Case H.1, the stream can not be heated to $T_{PI}$ by this heat surplus since $T_{PI}$ is pinch temperature for hot streams. The heat deficit is, thus, $mc_p\Delta T_{min}$ for heating the stream from $T_{PI}-\Delta T_{min}$ to $T_{PI}$. For Case H.4.b.ii, recuperative heating is not used. When $T_s < T_{PI}-\Delta T_{min}$, the heating of the stream from $T_s$ to $T_{PI}-\Delta T_{min}$ can be covered by the heat surplus below pinch. The heat deficit is, thus, $mc_p\Delta T_{min}$ for heating the stream from $T_{PI}-\Delta T_{min}$ to $T_{PI}$. When $T_s \geq T_{PI}-\Delta T_{min}$, the heating from $T_s$ to $T_{PI}$ causes a heat deficit of $ymc_p\Delta T_{min}$, where $0 < y \leq 1$.

Similar analyses can be performed for the other cases. It can be concluded that, when Pinch Expansion is used for a stream (independent of its type before and after expansion and the location of $T_s$ and $T_t$ on the GCC) and the portion is constrained to be $(mc_p)_{exp,PI,max}$, the hot utility demand is underestimated by an amount of $ymc_p\Delta T_{min}$ where $0 \leq y \leq 1$, when the GCC is used as a simple graphical tool as part of the novel targeting procedure presented in this article.

## Appendix B: Cost Analysis for Example 1

The total annualized cost (TAC) is calculated and compared based on the following assumptions: (1) Only capital cost (for heat exchangers and expanders) and utility cost are considered; (2) The plant lifetime is 30 years; (3) The operating time is 8000 hours per year; (4) the interest rate is neglected for both capital cost and utility cost; (5) the price of power, hot and cold utilities are 0.05, 0.02, and 0.001 USD/kWh, respectively; (6) the heat-transfer coefficient ($h$) is assumed to be 0.05 and 0.3 kW/(°Cm$^2$) for process streams and hot/cold utilities, respectively.[21]

For heat exchangers, the heat transfer area, $A$ (m$^2$) is then determined by Eq. B1[22]

$$A=Q/(U\Delta T_{LM}) \tag{B1}$$

where $Q$ is the heat transferred, $U$ is the overall heat-transfer coefficient, and $\Delta T_{LM}$ is the logarithmic mean temperature difference. The capital cost of the heat exchangers (2010 basis), $C_{HE}$ (kUSD), is estimated by Eq. B2[22]

$$C_{HE}=28+0.054A^{1.2} \tag{B2}$$

The capital cost of the expanders (2007 basis), $C_{EXP}$ (kUSD), is estimated by Eq. B3[23]

$$C_{EXP}=661+0.127W \tag{B3}$$

All costs are converted into the 2013 basis using the Chemical Engineering Plant Cost Index (CEPCI).[21] The cost performance for various cases in Example 1 is shown in Table B1.

**Table B1. Cost Performance for Example 1**

| Cases | A | B | C | D |
|---|---|---|---|---|
| Heat exchangers | | | | |
| H1C1 | | | | |
| Heat load (kW) | 108.8 | 214.5 | – | 174.6 |
| CAPEX (kUSD) | 1.811 | 1.798 | – | 1.196 |
| H1C2 | | | | |
| Heat load (kW) | 540 | 540 | 540 | 540 |
| CAPEX (kUSD) | 3.685 | 3.685 | 3.685 | 3.685 |
| C1C1 | | | | |
| Heat load (kW) | – | 200 | – | 355.4 |
| CAPEX (kUSD) | – | 3.613 | – | 6.247 |
| Heater C1 (before expansion) | | | | |
| Heat load (kW) | – | – | 160 | – |
| CAPEX (kUSD) | – | – | 1.317 | – |
| OPEX (kUSD) | – | – | 25.6 | – |
| Heater C1 (after expansion) | | | | |
| Heat load (kW) | 360 | 200 | 352 | 200 |
| CAPEX (kUSD) | 1.509 | 1.366 | 1.503 | 1.366 |
| OPEX (kUSD) | 57.6 | 32 | 56.32 | 32 |
| Heater C2 | | | | |
| Heat load (kW) | 540 | 540 | 540 | 540 |
| CAPEX (kUSD) | 2.388 | 2.388 | 2.388 | 2.388 |
| OPEX (kUSD) | 86.4 | 86.4 | 86.4 | 86.4 |
| Cooler H1 | | | | |
| Heat load (kW) | 371.2 | 265 | 480 | 305.4 |
| CAPEX (kUSD) | 1.419 | 1.322 | 1.501 | 1.361 |
| OPEX (kUSD) | 2.970 | 2.12 | 3.84 | 2.443 |
| Expanders | | | | |
| Load (kW) | 308.8 | 255 | 351.9 | 214.6 |
| CAPEX (kUSD) | 25.334 | 25.065 | 25.55 | 24.863 |
| OPEX (kUSD) | 123.52 | 102 | 140.76 | 85.84 |
| Total CAPEX (kUSD) | 36.145 | 39.237 | 35.944 | 41.107 |
| Total OPEX (kUSD) | 23.45 | 18.52 | 31.4 | 35.003 |
| TAC (kUSD) | 59.595 | 57.757 | 67.344 | 76.11 |

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